# From lecture to active learning: Rewards for all, and is it really so difficult?

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In the centuries-honored *I-You* paradigm, a mathematics instructor provides first contact with new material via lecture, then expects students to solve problems outside the classroom: *I* lecture, then *You* do homework alone[1]. Twenty-five years ago I began a personal journey away from this paradigm, leading to a pedagogy that today fits within what is broadly termed active learning. I will provide an analysis of my decades-long evolution to a very particular non-lecture teaching philosophy. My experiences provide encouragement for readers who may worry that alternatives to lecture are complicated and time-consuming. The main message is that it needn't be difficult to create active learning for your students, and that there are tremendous rewards for the instructor as well as for students. My goal is to entice hesitant readers to take a teaching plunge.

Through the years my experiences convinced me that what can happen in an active learning classroom can be greatly enhanced by good student preparation before class. I expect students to prepare via reading, writing, and problem work. Classroom activity can then build directly on their preparation, with plenty of in-class feedback from fellow students and the instructor. Together, these components—student preparation and active classroom learning—enable each student post-class to tackle higher level homework. I call this integrated, multi-pronged approach my "ABC method", a paradigm that I have refined in sixteen different courses[2] at all undergraduate and graduate levels. See `https://www.math.nmsu.edu/~davidp/` for lots of assignment examples, detailed guidelines for students and for grading and daily logistics, and a holistic rubric.

In a nutshell, to be described in detail later, my evolved paradigm for student work is:

- A, *due well before class:* Read, write questions, respond to my questions. Graded for completion only.

- B, *bring to class*: Prepare 'warm-up problems'. Graded for completion in advance only.

[1] The *I-You* terminology is discussed in [8].

[2] Lower division: Calculus I,II,III, Discrete Mathematics, Discrete Mathematics for Computer Science, Mathematics Appreciation, Spirit and Evolution of Mathematics.

Upper division: Abstract Algebra, Real Analysis, Combinatorics, Topology, Geometry, Number Theory, Great Theorems.

Ph.D. level: Topology I,II.

- *During class*: Directly build active learning on both A and B.
  - Lead brief discussion of the questions from A.
  - Compare and complete problems from B in groups: I facilitate, students present.
- C, *after class*: A very few harder 'final problems' building on B from class. Marked carefully, may be asked to redo to perfect, holistic letter grade only. Final level of achievement.
- ABC a very large part of course grade; harmony between learning and evaluation, reduce exams.

I will share my personal thinking that led to this paradigm, its nuts and bolts, what has and hasn't worked, in which courses, how issues of time and coverage work out in practice, and student actions and reactions.

Equally important, since active learning likely seems shaky ground for those primarily used to lecturing, I will provide some needed reassurance about how demanding and time-consuming the shift I made is or isn't, and what the pitfalls are. I will also address inertia, challenges, efficacy, teaching materials, burnout, buy-in, and rewards, not only for students, but at least as critically important and motivating, for instructors, since rewards for instructors are perhaps crucial to overcome hesitancy.

In the intervening decades since my own evolution began, an enormous body of research has developed which concludes that there are better alternatives than the *I-You* pedagogy for student success, collectively termed active learning. Moreover, active learning confers disproportionate benefits for STEM students from disadvantaged backgrounds and for female students in male-dominated fields. And these benefits accrue while not disfavoring high-achieving or more experienced students, or any demographic group [2, 5, 14, 23]. Recently the presidents of fourteen professional mathematics societies joined to exhort us to shift from *I-You* toward active learning [4]. The question, then, is with what exactly, and is that hard to do?

All active learning paradigms[3] share two in-classroom features. First, reduce or eliminate lecture. Second, devote substantial classroom time to student involvement in mathematical work that receives immediate feedback from other students and from the instructor. These involve more of the ingredients *You* and *We*, and considerably less of *I*. Concomitantly, students will be more in charge of and responsible for their own learning, while instructors will have increased responsibility to guide student work. But within these broad parameters, many variations are possible, so this is an exciting time of great experimentation by many people to seek out and compare a variety of good active learning teaching techniques for mathematics. We already have a wonderful resource in [17].

What I can contribute is one independently developed example, the evolutionary process of my own questioning, teaching experiences, reflection, and adaptation leading to a philosophy, a paradigm, and a personal long-term implementation within the broad framework

[3]E.g., inquiry based learning [10, 13], interactive engagement [5], total quality management [24], just in time teaching [9, 19], peer instruction [18], flipped or inverted classroom [1], process oriented guided inquiry learning [20].

of active learning today. The distinguishing feature of my evolved paradigm is its special emphasis on tightly integrating pre-class preparation, via reading/writing and problem work, with in-class active learning and post-class follow-on homework.

## Lecture

From my early life as an *I-You* student, I remember occasional inspiration from lectures, but there was not much learning there that enabled me to complete anything but rote homework. After all, lecture usually primarily involves the instructor demonstrating that s/he can do the mathematics. But my teaching experiences have led me to believe that this rarely helps a student actually be able to do much mathematics, any more than a swimming instructor demonstrating an hour of beautiful swimming techniques successfully teaches a beginner how to swim various strokes. As a student, I survived and prospered despite a lecture setting, only by reading text material over and over integrated with tackling homework challenges. I now realize this was essentially autodidactical, my instructor's role chiefly being to provide a schedule, expectations, homework feedback, and evaluation via exams.

My subsequent decades teaching thousands of students suggests that few students will very successfully self-teach as above. The paradox for readers of this article is that we are probably the most notable group of exceptions; we are among the rare survivors or "thrivers" of the *I-You* approach. But I expect we all have frequent conversations with random adults, and with colleagues from other disciplines, all former *I-You* students of mathematics, in which we receive strong unsolicited confirmation from them that the average *I-You* experience was a dramatic failure leaving many scars.

During the years I lectured, many students told me "I know the math. I understand perfectly when you lecture, but then I can't solve problems at home." Of course in actuality this meant they didn't really "know the math", but I didn't know what I could do to help, other than to lead them through homework problems. In retrospect, for all but possibly inspiration or rote learning, my lecturing was ineffective, despite all my best efforts, and notwithstanding my students' encouraging lauding of my lectures, their desire for it, and belief in it. And since it wasted precious classroom time, it was inefficient as well. In fact, classroom lecture may well become largely obsolete, since with modern technology any recorded lecture can be viewed by anyone, anytime, anywhere. How long will it take university administrators to conclude that they need not employ professors to add more lectures to the increasing number already archived? In short, professors had better have something more to offer students than yet more lectures on settled subjects [3, 11]. Of course we will all claim that our students really do need much more than a lecture to succeed, and that we can provide that. So isn't that what we should home in on? How then do we both challenge students and guide and support their work as learners in truly productive ways?

## First contact with new ideas

In rethinking the *I-You* paradigm, much revolves around the question of "first contact"[4]: How and when should a student first be exposed to new material? In mathematics especially,

[4] I was enormously influenced by the ideas of Barbara Walvoord [21, pp. 53–63] on first contact with new material, as beautifully described to me by Virginia (Ginger) Warfield.

absorbing and making sense of substantial new ideas with any depth is usually a slow, highly individualized, intellectually messy business. Lecture is by nature time-limited, one-size-fits-all, and totally incompatible with the need to "Stop, wait a minute, let me think that through and pose a question." In short, lecture is on its face a poor means for first contact with demanding new material, despite our natural inclination to the contrary, that as instructor we can help students get started digesting new ideas by offering them a lecture first.[5]

So if in-class lecture provides poor first contact, then perhaps first contact, and maybe even first problem work, might better occur *before* class, and something entirely different can happen *during* class. This could lead to the recently named "flipped" or "inverted" classroom. In its original conception, lecture and homework switch venues, with students watching recorded lectures before class and working together on mathematics during class. But watching recorded lectures has several drawbacks too. It is in many ways at least as passive as watching a lecture live. And it suffers the same cognitive drawbacks given above for in-class lectures, unless a student were to frequently hit pause and replay, all the while thinking things through deeply and asking questions, which is unlikely to happen without additional pedagogical structure. Finally, the amount of instructor time and effort needed to create recorded lectures is enormous, unless one utilizes someone else's.

My own conclusion, arrived at twenty-five years ago before recorded lectures were even practical, was to evict lectures entirely and evolve new paradigms instead. This shift was actually catalyzed by my NSF-funded team's development of student calculus projects [6]. When we decided to allocate some time for student groups to work in the classroom on their projects, with instructor help, lecturing had to be reduced, and I began to expect students to read new material before class instead. It is extraordinary to realize that my shift might never have occurred absent this external force.

## Reading and the lecture-textbook trap

How then do I want students to obtain meaningful first contact with new material before class? My simplest answer in most courses is for students to thoughtfully engage high-quality reading. And yet, while reflection and thinking stimulated by reading can be extremely powerful, simply exhorting students to read the book before class rarely works, since they seldom read as suggested.

There is a gaping trap here, a truly vicious cycle in which students don't read beforehand when they know the instructor will lecture, and instructors lecture in large part because they know students haven't read. Breaking out of this lecture-textbook trap was the most difficult teaching problem I ever had to solve, but all else flowed from it. I felt it was my responsibility to break this cycle by insisting (to self and students) that I will not lecture, and instead arranging for in-class activity to be built on a foundation of high-quality student preparation. A guiding motto was born: "Never lecture on something students can read instead."

[5] For more on the disconnect between the role of lecture for instructors and students, see [22].

## Written response to reading: Part A

Resolving the lecture-textbook trap was the continuing catalyst for my entire journey, enabling me over a number of years to evolve completely away from lecture.

An obvious resolution to the trap was to somehow convince students to read in advance. However, seemingly making the trap even worse, reading alone is insufficient, even if I convince students to do it, since they won't get much our of merely reading. For two reasons, I realized I needed to guide students to reflect and think critically about what they read, to make connections, and to respond in writing well before class. First, the nature of an intellectually challenging writing task should hone their thoughtful engagement and critical thinking and analysis, thus making the reading worthwhile for learning. Second, I really need their written responses to reading in order to prepare myself for the next productive non-lecture class session.

I have never asked students to summarize written material. Rather, I have always challenged them to engage deeply by coming up with thoughts on their own. For lower-division students, I find that written response to a couple of well-crafted reading questions from me is essential. The questions I ask are designed to stimulate students to read and think carefully, and to catalyze and help guide class discussion. And sometimes my reading questions aren't questions at all, but brief tasks based directly on the reading. I also expect all students at every level always to write their own good mathematical questions about their reading, and additionally to write which new concepts are confusing, what was well explained and interesting, what they had to reread but eventually understood, and what connections they see to other ideas. Textbook reading tends to provide polished answers to questions not even meaningfully asked, and I attempt to get beyond this by expecting regular good mathematical questions from students.

As examples of reading questions I might pose, in a first calculus course, after reading a textbook section introducing the derivative, I might ask: "What are the different mathematical and physical interpretations we know of for the derivative of a function?" and "Explain in your own words what your understanding is of the idea of the derivative of a function." Or, in a discrete mathematics and introduction to proofs course: "Make up two great examples of your own of multiply quantified statements, in which the meaning changes dramatically when the order of the quantifiers is changed as in Examples 2.2.1 and 2.2.2. Explain why this is the case for each." and "Make a good example of your own of each of the two types (existential and universal) of multiply quantified statements discussed, and then write and explain their negations." Or, for the beginnings of a logic project based on primary historical sources [12]: "What is logic?", "What did Boole attempt to create?", "What is an 'implication'?", "How are implications related to modern computers?", "According to Aristotle, what is the difference between a sentence and a proposition?", "What is a syllogism?" More examples of reading questions are available at `https://www.math.nmsu.edu/~davidp/`.

My experience is that students will commit seriously to reading and writing provided they both experience the benefits and know it is highly valued, i.e., in class and in their grade. I mark each reading/writing assignment very quickly, holistically, with a single +,✓,- grade, only for seriousness of effort. I make as many or as few comments as I wish or have time for, requiring only about five to fifteen seconds per paper, since I am never reading detailed mathematics. My greatest marking intent is to make sure that each student sees that I have read and thought about what they wrote. Students become very faithful to this reading and writing, and although I expect less than half a page of response, some students become so emphatic about its benefits that they insist on writing more, whether I want it or not! Some even explicitly credit their success in the course to this activity.

If I receive student written reading responses up to one class period beforehand, on paper or electronically, I can read them and determine how my students are reacting to the new material. This best prepares me to guide class without even a nagging impulse to lecture. I spend no time preparing a lecture, rather I prepare notes on their writing so that I can best guide their learning in the classroom.

Does this require reading material different from a textbook? Not necessarily, provided the reading is genuinely accessible, interesting, stimulates provocative thinking and questions about new ideas, and provides good grist for class discussion. So I choose reading materials carefully for these goals, possibly utilizing multiple materials with different points of view to compare. This does not mean that I choose material that promises to make the subject "easy" or a "straight path", since such features often mean that the challenges, questions, interest, and depth are missing, which does not serve learning in the long-term.

## In-class discussion of reading/writing: Building on Part A

Class can now begin with a discussion directed by me, based on the few notes I made while reading students' responses, which I first return with any comments. It is always focused just on their writing, instead of a shoot-in-the-dark lecture trying to address everything, without knowing what students are struggling to understand.[6] It can be geared specifically to meet their needs for understanding the content I asked them to read, and the discussion leaves no need for me to even consider lecturing. I sometimes have students read out loud selected questions of theirs, and ask for class reaction. This keeps the discussion focus on them and their thoughts, not largely on mine. Since students have thoughtfully engaged the reading, this second-contact in-class discussion never needs to be lengthy, usually 5-15% of class time, and the vast majority of time is available for something else. What now could most usefully happen in class?

## Warm-up problems beforehand: Part B

Auspiciously, student written response to reading has prepared them for productive initial mathematical problem work. So why not assign easy-to-medium-difficulty "warm-up" problems, also to be prepared in advance, and brought to class? Then in class these problems can be compared, discussed, presented, and completed, using the vast majority of classroom time, so that by the end of class the level of student mathematical accomplishment, and their confidence, is high. I imagine a traditional homework assignment, and put all but a few hardest problems into the warm-up problems to be prepared for class. In terms of amount, it should be just enough to keep everyone at work for the full class period, but not too much

[6] Lecture has a shoot-in-the-dark character because it is a one-way communication; its etymological origin is 'to read a discourse aloud'. It is interesting that the 'co' in communication (etymology 'sharing', Latin *communis*, 'common') seems absent in lecturing, so perhaps lecture doesn't deserve to be called communication.

more, since I want everyone to feel that this assignment is satisfactorily completed by the end of class, and they are ready for a final homework assignment of the harder problems.

My experience is that students will faithfully prepare warm-up problems in advance of class, provided they directly experience the benefits, know it is highly valued, and be fully expected to contribute in class based on their preparation in advance. This has been so successful that I have never had a single student express reluctance about doing this homework as preparation before, rather than after, class. The learning benefits quickly become obvious to students, since by the end of class time, they are confident they have solved the easy-to-medium warm-up homework, and feel ready to tackle harder problems at home.

## In-class active learning: Integrating and building on Parts A and B

The stage has been pre-set for classroom active learning by student preparation in advance, consisting of both reading/writing and warm-up problems. When I arrive at the appointed hour, I find most students already comparing their prepared work in small groups. We begin together with a brief discussion based on their reading/writing, as described earlier. Then most of class time is spent in small group work refining their warm-up solutions, and in board presentations, as I continually circulate.

My aim is to interact personally with every student or group multiple times. My own classes have ranged from 10 to 50 students, with no lecture/recitation format. Even in a class of 50, I am usually able to interact personally and substantively with every student at least once during a 75 minute class period that meets twice per week. I keep on the move, staying with each student or group just long enough to provide encouragement, a little advice, and to learn what they are struggling with. If a student or group is stuck, I stay just long enough to help on that point, then let them continue by themselves rather than rely on me for further straightforward progress. If not stuck, then I may offer encouragement but not stay long, unless a significant opportunity for enrichment presents itself to be seized. I am also always thinking about what I should do next. Should I select a student to put a certain problem on the blackboard, or should I initiate a whole-class discussion on a particular problem, or go on to another student or group? Compared to this, lecturing is straightforward.

I spontaneously initiate either whole-class discussions on particular problems, or individual student board presentations and discussion. Often I will ask several students to write solutions to various problems on the board simultaneously, and then we discuss them all at once as a class. Sometimes a writer is asked to verbally explain what has been written, sometimes not. Not every problem necessarily gets presented or discussed.

I discovered painfully once, in a calculus course, what happens if I steal students' in-class work time by lapsing into lecture. I thought that the material for the day was particularly tough, and that if I began with a bit of lecture, it would help. After a while I saw frustration on my students' faces, and I realized my mistake: They wanted to get to work on what they had prepared for their valuable in-class time together, not listen to me. I now realize I should be very happy about that; they are in charge of their learning, and they know it and own it.

I am generally laissez faire about how groups arrange themselves. Three issues with student group work are of concern. If the rare individual prefers to work alone, I repeatedly encourage them to work with others, but I will ultimately not force it. And if a group is not functioning optimally, I will sometimes ask a member to switch to another group, to see if the result if better. Finally, I may occasionally reshuffle all the groups.

What are the forces ensuring that students really prepare problems before class? Partly it is group peer pressure, and subtle pressure by me circulating to observe student prepared work, and also the certain knowledge that I may ask any student to present a prepared problem on the board at any time. I make my presentation choices spontaneously, but very carefully and consciously, including to apply pressure for better preparation beforehand by individuals if necessary. But students' main motivation to prepare is their experience that it creates a very effective learning environment, one in which they will end class well equipped for the final, harder, after-class homework.

The warm-up problems are collected at the end of class, and marked holistically +,✓,-, again strictly for seriousness of effort at preparation in advance, and they are important in the grade. I could alternatively have an extra copy due at the beginning or before class, e.g., photograph/scan and submit online. Since the warm-up problems are dissected in class, I never read them individually. I am interested solely in whether the student prepared them in good faith before class. This literally takes only five seconds per paper. Even though I am collecting them only at the end of class, it is not hard to train myself to instantly assess preparation in advance. This is particularly easy if, as often happens, there is a warm-up problem that we didn't get to in class discussion; then I can easily see on each student's paper whether the problem was prepared beforehand or not.

## Final problems after class: Part C, exams, and course grade

With student preparation before class of reading/writing and warm-up problems, all in support of in-class discussion, group work, and presentation, it remains only to assign a very few (two or three) harder "final" homework problems for students to complete after class. These are like the hardest few of traditional homework, but now build on what students have already achieved before and during class. The final problems are the only daily work needing detailed marking, representing each student's highest level of achievement on the day's material.

Their papers receive prompt and very careful feedback, a single holistic letter grade[7], and possible prompt redoing of individual problems at my initiative to bring to perfection. They are normally never discussed in class. These higher level problems are at the core of a student's course grade; I consider them the best measure of what each student has learned and accomplished. The message to students is that their three daily written components are the fundament of both learning and evaluation, so I find it critical that they form the vast majority of the course grade. This leads to a reduction in exams, which I believe is good, because my experience is that timed in-class exams are a poor way for most students to demonstrate what they can actually do.

[7] I have long found using points in marking to be a time-wasting, exhausting, distracting, and deceptive morass that sends the wrong message to students and invites trouble. While I may write a lot on student papers, I always assign only a single holistic qualitative evaluation to a paper, be it homework or an exam.

I have always made the three ABC components together of student daily work at least 60% of the holistically assigned course grade, with the carefully graded final problems (Part C) dominant in my mind. Since almost all of the reading/writing (A) and problem preparation (B) assignments earn a +, the harder, after-class, final problems (C) with letter grades tend to become paramount in each student's grade. In some courses I still use some form of one midterm and final exam, while in others, especially upper division courses, I sometimes use no exams, since the daily Part C graded work with high expectations is more than sufficient for evaluation. Usually I clearly leave open the option for exams, and often decide against them as the term progresses.

A word of warning based on experience: Once upon a time I didn't clearly separate the warm-up from the final problems, but this led to complications, including lesser student effort on the warm-up problems before class; I find it works far better with the two sets well separated.

How can one be sure that these final problems completed outside class represent the work of each individual, especially since I encourage students to work together in class so much? Truth be told, even on the final problems I make it clear that students may discuss them together. My ironclad rule, though, is that when they go to write them up, this must be done alone, so that no two papers should look alike. Since these harder problems are never just a calculation or formulaic, but always involve explanation of ideas, I can detect not only the level of understanding, but also easily observe if two papers are too similar. If so, which occasionally happens at the beginning of the course, I speak with the students involved to reiterate my expectations as emphatically as necessary. Another issue could be students finding and copying solutions from elsewhere, e.g., online, to problems for Parts B or C. Since this completely defeats their purpose, if this is a danger one must create or tweak problems to avoid it.

## Précis of student assignments

To sum up the evolved paradigm, which tightly integrates before-, during-, and after-class work, students write three homework papers for each daily unit of content (in my case twice weekly), which I call parts A,B,C, and which replace the *I-You* paradigm:

- *You*, Part A: Read/write, received up to one class period early for me to prepare for leading class discussion. Marked quickly +,✓,- for effort only.
- *You*, Part B: Warm-up problems, prepare and bring to class. Marked quickly and holistically +,✓,- for preparation only. Submit in person, and/or online photo/scan before class.
- *We*: In-class discussion, group work, presentations, all built on Parts A,B for the given unit.
- *You*, Part C: A very few harder final problems, completed after class and written up alone. Marked carefully with feedback and holistic letter grade, sometimes specific problems redone at my request.

- Together Parts A,B,C constitute the majority of the course grade, reducing or eliminating exams.

Note that for a given unit, Parts A,B,C are due at different times, so there is a rolling nature to the coverage of multiple units. Students easily adapt to this provided I give clear guidance, and it has integrative benefit.

## Inertia

There are many forces binding an instructor to the *I-You* paradigm, even if open to change. First, we naturally tend to teach as we were taught. Second, a lecture "covering material" is in the tradition of fulfilling professorial duty. It is hard to let these go, and to realize that instructor coverage does not necessarily help students do mathematics. Students, too, are mostly happily complicit, generally very comfortable with passive receipt of a lecture. Certainly it is much easier than having to do any actual work in the classroom, and they can believe they must have learned something from lecture.

Third, it takes real effort to change pedagogy, and will likely catalyze a process of further evolution, so it is a major commitment. Fourth, there is an element of uncertainty, worry, and perhaps fear of classroom disaster. Lecture is well-known, often easy, predictable, and contains essentially no element of risk, since it is totally controlled by the instructor based on preparation in advance, with little chance of something unexpected or surprising from students. On the other hand, a classroom of continual interaction with students, reacting to, adjusting, and guiding what students initiate may seem scary. Moving away from lecturing amounts to relinquishing total control, but hopefully without totally losing control, since one still has overall guiding responsibility. Creating the right balance is a challenge.

Therefore, since shifting from *I-You* requires overcoming much inertia, it will most likely occur only if one sees large benefits and rewards, and not too many scary challenges, not only for students and their learning, but also for instructors themselves, and here I would like to offer much encouragement from experience. Let's begin with the students.

## Benefits for students and learning

My experience is that students respond well to preparatory work provided they quickly and consistently experience the advantages. They then find their in-class work time valuable, engaging, rewarding, often exciting, and confidence-building. Completing the warm-up problems with feedback from me and fellow students in the classroom prepares them well for success with the few final, harder problems to be completed after class for careful grading, and they know and greatly value that. Many times my students are so absorbed in group work completing warm-up problems that they don't realize when class time has ended. I apologetically interrupt the whole class to tell them that class ended five minutes ago! When does that ever happen with a lecture?

The reduction of exams along with the predominant emphasis on daily work for both learning and course grade creates a much steadier workload for students, yielding the cognitive advantages of spaced learning, and relief from the typical exam/cram/forget phenomenon that doesn't foster long-term learning. This also places learning and evaluation in harmony, reducing stress and producing more consistent quality of work. My impression from a many-years evolution is that, with these approaches, my students work more, and more successfully learn course material.

Student course evaluation comments are almost uniformly positive about the pedagogy, and indicate a high level of buy-in. They typically credit preparation in advance for in-class collaborative work as extremely effective for their learning, and for keeping them on top of the course with less stress. Students also often remark that the emphasis on student participation makes the subject come alive. And quite frequently they ask why other mathematics courses are not taught this way.

I mention here one anecdote that still astonishes me, from an abstract algebra course intended both for mathematics majors and future secondary mathematics teachers. Although the entire course was focused on mathematics, at the end of the semester one student came to my office to tell me that for her, more important than the mathematics had been the teaching style, and that she had consciously spent the entire term studying the pedagogy, with the aim of adapting it in her own teaching. Never had I dreamt that while thinking I was teaching abstract algebra I was actually also teaching pedagogy.

## Challenges for instructors

Shifting from *I-You* to something like *You-You-We-You* has initial challenges for an instructor. As with anything new, more effort will be needed the first time. Experience pays off handsomely, though, and after once or twice through, my experience is that the overall workload should be no greater than for *I-You*.

It is critical that students have confidence from the start. I build this by briefly explaining to the class the evidence for how an active learning paradigm will enable them to be successful in the course, that class time will be interesting, productive, and satisfying, that it will prepare them well for the harder homework, and that this daily work is the great majority of their course grade. And I assure students that I will be there to give personal help in class every day. Then I watch and listen to how things are going, especially in the first weeks, repeat my explanation for active learning as necessary, and take prompt steps to resolve any confusion and alleviate discomfort.

One must learn, as addressed above under In-class Active Learning, how mindfully to make decisions that support student learning in a less predictable classroom environment where control and responsibility is being loosened and partially handed to students. I also need to keep reminding myself that in a nonlecture classroom, it is students who should be doing the mathematics, not the instructor, since I already know the mathematics, and they are the workers and learners. My job really should be that of effective, efficient, encouraging, and hopefully inspiring, guide and manager.[8] Neither should anyone expect their learning to be easy: I can be helpful in many ways, but the learning is their work, just as when Euclid is said to have replied to King Ptolemy's request for an easier way of learning mathematics that "there is no royal road to geometry".

Perhaps the greatest danger for an instructor is that with students handing in homework

[8] Frank Williams was a seminal influence for me, for which I am forever grateful. It was he who helped me understand the proper role of the instructor to complement the student's role of worker and learner [24].

Parts A,B,C for each class day, it would be all too easy for me to do way more homework marking than I should, and therefore spend more time teaching this way; I have witnessed colleagues insistently fall into such a hole when trying this approach. Each of the three parts is crucial for student learning, but Parts A and B do not need grading or instructor feedback on the mathematics, since this all happens in class. While Part C is carefully marked, and perhaps pieces redone, it consists only of two or three harder problems, making grading manageable.

I am often asked how to start on the first day of a term, since Parts A and B are to be completed before class. First, I never lecture; instead I model the pedagogy on that first day by having students work together on meaningful mathematical activity. Then between the first and second class days, I have students submit their first response to reading, the only time anything happens off schedule; thus we are ready for a normal routine on the second day of class, when the first warm-up problems and the second response to reading are due.

Is my paradigm a one-size-fits-all approach? While I have found the basic components to be universally successful, the details may best differ between courses at different levels, or with different meeting schedules, or with large size classes.

For instance, in a mathematics appreciation general education course at the lowest college level, I emphasize hands-on activity more than reading, for both work at home and in class; and after-class work often entails students just writing up what they discovered in class. At the other extreme, in a Ph.D. level graduate course, I often ask students to contrast multiple different written approaches, and in class I will ask students to present their own versions of proofs at the board and lead discussion thereof. Regarding exams, I already discussed above, in Final Problems After Class, how I have reduced exams differentially in courses at different levels.

Regarding class meeting schedules, in the past couple of years I have worked with a number of faculty and graduate teaching assistants who are adapting this pedagogy to different class meeting schedules than mine, which is two 75-minute class sessions weekly. I see that the scheduling of Parts A,B,C may best be adjusted, with some consolidation in a course with three or more meetings per week, e.g., only two Parts A and B weekly, and/or one Part C. Some adaptations are very innovative and substantial, such as in [7].

And for large class sizes, or courses traditionally scheduled in lecture/recitation format intended specifically for lecturing, I see other successful adaptations being made, e.g., with one or multiple graduate teaching assistants and/or undergraduate learning assistants [15, 16] present in the classroom with the instructor to work with students.

Finally, what must I at minimum have within my control in order to teach this way? I need my students to have access to good reading and problem material that I can assign as needed, including reading/writing and problem preparation in advance of class. I need the daily pre- and post-class assignments to be the core of students' work and grade. And I need to be able to mold the classroom environment into an active one and gain the confidence of my students. All else is flexible.

## Benefits and rewards for instructors

Perhaps for many instructors, at the end of the day it will also be the personal rewards, not just those for students, that will seem attractive about alternatives to *I-You.* I admit I have reaped tremendous personal rewards.

Class time has higher quality interactions and is more exciting when one is frequently discussing interesting mathematical ideas with individuals and groups, and they are coming up with questions and ideas and points of view that one hadn't anticipated. The enthusiastic response from students is extremely gratifying, as is the learning success one sees. In short, I enjoy interacting with my students much more, a huge benefit!

Marking student work is more rewarding in two senses. First, exams are fewer. Second, marking time is spent primarily on the few harder homework problems, which are more interesting to mark, not on the easier material that has been dealt with in class. And the remaining time is spent mostly reading student responses to reading, which stimulates and prepares one with confidence to lead a class discussion most useful for student learning.

Time, ah time: My experience over many courses is that an alternative to *I-You* need not take more instructor time overall, provided one does not fall into the trap of unnecessary over-marking of student homework prepared for class. A perhaps surprising timesaver is that students often need less of my time in office hours: By replacing lecture with student interaction with each other and with me on active work inside the classroom, students get most of the help they need, and their questions answered, in class. Moreover, the steadier workload mentioned above applies to instructors as well, so there is very little end-of-term stress, and no long-term burnout.

With rewards as strong as these, I could never return to *I-You*. Carpe diem!

## Is there really an elephant in the classroom?

Finally, consider the question of coverage, an intimidating and much-feared elephant. When I talk with *I-You* instructors about replacing lecture with student work in class, they almost invariably reply "But then I couldn't cover all the material in the syllabus". My primary answer, of course, is that it is not the instructor who needs to cover the material, but rather the student.

I have found, in teaching many types and levels of courses, that if high quality first contact and initial mathematical work happens before class, thus making lecture irrelevant and redundant, and if class time is instead used for student work with others and with the instructor to build on the work prepared in advance, then coverage is always more efficient, not less so. To me this simply makes logical sense: If lecture is a largely ineffective use of precious classroom time for student learning, then offering students a guided active-learning classroom environment, working with each other and with me, seems likely to proceed more efficiently, especially when first-contact reading and preparatory work happens before class. Specifically, I have taught this way in first year calculus courses with multiple sections all following the same lockstep routine with common exams, where students in my section had to progress at the same rate that other instructors were lecturing, and this was no problem at all. In fact it was in exactly that setting, with a class of 45 students and no grader or teaching assistant, where I first developed and refined the approach described here.

My consistent experiences after transforming *I-You* into *You-You-We-You*, in many courses at all levels and for all college audiences, is that the content is actually less rushed. I found no fearsome coverage elephant in the classroom as I redesigned it, even with the same syllabus as other instructors.

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