Photo 5.6 Albert Magna, original 1475 edition.

He is described as a well-educated man with a deep interest in mathematics and science. He has written extensively on these subjects. At the age of 20, he became a professor at the University of Bologna, and later moved to Padua, where he taught mathematics and astronomy. His work on geometry and algebra was groundbreaking.

Figure 5.6: Proposition 6.

Figure 5.7: Proposition 11.

Now complete the proof of Proposition 11 in the spirit of Euclid.

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The Cube of a Cube from Cipriano Cardano, from "De Arithmetica," Chapter XI of the Ars Magna, (The Great Art), the Element 6. The term "the two mean were of equal difference, arithmetic mean 6, geometric mean 8, 6, 8") is derived from Cipriano Cardano, from the Ars Magna, (The Great Art), the Element 6. His method of solving cubic equations was to use a geometric approach and to find the roots of cubic equations by using the Greek method of geometrical constructions. In his work, Cardano used the concept of the "cubic root" to solve cubic equations, which was a significant advancement in algebra. He also introduced the concept of the "cubic equation," which is a polynomial equation of degree three. This work was instrumental in the development of algebra and laid the foundation for modern algebra.
It remains to be shown that \(3AC(AB \times BC)\) is equal to the six bodies. This is clear enough if I prove that \(AB(BC \times AC)\) equals the two bodies \(AB \times BC^2\) and \(BC \times AB^2\), for the product of \(AC\) and \((AB \times BC)\) is equal to the product of \(AB\) and the surface \(BE\) — since all sides are equal to all sides — but this [i.e., \(AB \times BE\)] is equal to the product of \(AB\) and \((CD + DE)\); the product \(AB \times DE\) is equal to the product \(CB \times AB^2\), since all sides are equal to all sides; and therefore \(AC(AB \times BC)\) is equal to \(AB \times BC^2\) plus \(BC \times AB^2\), as was proposed.

Rule

The rule, therefore, is: When the cube of one-third the coefficient of \(x\) is not greater than the square of one-half the constant of the equation, subtract the former from the latter and add the square root of the remainder to one-half the constant of the equation and, again, subtract it from the same half, and you will have, as was said, a binomium and its apotome, the sum of the cube roots of which constitutes the value of \(x\).

For example,

\[x^3 = 6x + 40.\]

Raise 2, one-third the coefficient of \(x\), to the cube, which makes 8; subtract this from 400, the square of 20, one-half the constant, making 392; the square root of this added to 20 makes 20 + \(\sqrt{392}\), and subtracted from 20 makes 20 - \(\sqrt{392}\); and the sum of the cube roots of these, \(\sqrt[3]{20} + \sqrt[3]{392} + \sqrt[3]{20} - \sqrt[3]{392}\), is the value of \(x\).

Again,

\[x^3 = 6x + 6.\]

Cube one-third the coefficient of \(x\), which is 2, making 8; subtract this from 9, the square of one-half of 6, the constant of the equation, leaving 1; the square root of this is 1; this added to and subtracted from 3, one-half the constant, makes the parts 4 and 2, the sum of the cube roots of which gives us \(\sqrt[3]{4} + \sqrt[3]{2}\) for the value of \(x\).

To the modern reader, Cardano’s “demonstration” of the subsequent “rule” seems cumbersome, because of the use of geometry. There are many references to Euclid’s Elements throughout the Ars Magna. Later mathematicians, in particular Viète and Descartes, developed a more efficient system of algebraic notation, completely freed from geometry (see, for instance, [22, pp. 84 ff.]).

What Cardano is proving in his demonstration of the rule for the equation \(x^3 = cx + d\) is that it is sufficient to find quantities \(u\) and \(v\) such that \(u^3 + v^3 = d\) and \(3uv = c\). Then the solution is \(x = u + v\). His proof becomes quite transparent if one uses a cube with side of length \(x\), subdivided into eight “bodies,” as indicated in Figure 5.9. From such a subdivision one can immediately deduce the Binomial Theorem for exponent 3, namely

\[(u + v)^3 = u^3 + 3uv^2 + 3u^2v + v^3.\]

FIGURE 5.9. Cardano’s cube.

This is none other than the “first proposition in Chapter VI” which he refers to. The main part of his argument is intended to show that

\[3uv^2 + 3u^2v = cx,\]

which is straightforward to reconstruct from the subdivided cube. Once he can show this, the desired formula follows easily, since he now is reduced to solving the system of equations

\[u^3 + v^3 = d,\]
\[3uv = c.\]

Its solution readily leads to the desired formula for \(x\):

\[x = \sqrt[3]{d/2 + \sqrt{(d/2)^2 - (c/3)^3}} + \sqrt[3]{d/2 - \sqrt{(d/2)^2 - (c/3)^3}}.\]

(Why does this formula seem different from the one at the beginning of the Introduction?) Of course, if the expression under the square root in the formula is negative, that is, if \((d/2)^2 - (c/3)^3 < 0\), then one faces the awkward problem of not being able to evaluate the formula. This might not be so bad if it were to happen only with equations that have no real roots. But evaluation of Cardano’s formula for the example

\[x^3 = 15x + 4,\]

which has the solution \(x = 4\), leads to the expression

\[x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}.\]

Basically, Cardano was at a loss how to deal with this case and dismisses it as absurd and useless in Chapter 37, where he deals with negative square roots [29,
The second half of the text continues from the previous page, discussing the development of mathematics and its applications. There is no clear mention of the mentioned code snippet or the search for an elusive formula as indicated in the image.