Project Report on

Dynamic analysis of Space Launch Vehicle coupled with fuel slosh and synthesis of control law based on Sliding Mode Control

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1. Abstract

The report studies the dynamic behavior of the space Launch Vehicle (LV) with and without liquid fuel sloshing. Three plant models (Launch vehicle) are developed which includes Geo Synchronous Launch Vehicle (GSLV) of Indian Space Research Organisation (ISRO). The plants are modeled from the dynamic equation of motion obtained from Lagrangian-Euler method in which the sloshing is represented in mechanical analogy as simple pendulum. Based on the dynamic behavior of the Launch Vehicle, a control law is synthesized to compensate the effect of influencing slosh dynamics.

2. Nomenclature

\begin{align*}
  m & \quad \text{Mass of sloshing liquid in kg} \\
  m_0 & \quad \text{Sum of rigid body mass and the mass of non-sloshing liquid in kg} \\
  v_x & \quad \text{Component of velocity in Xb} \\
  v_x & \quad \text{Component of velocity in Zb} \\
  l & \quad \text{Length of equivalent pendulum in m} \\
  b & \quad \text{Distance of pendulum hinge point from cg in m} \\
  g & \quad \text{Acceleration due to gravity in m/ s}^2 \\
  T & \quad \text{Control thrust in Newton} \\
  c_z & \quad \text{Force coefficient along body z-axis} \\
  c_x & \quad \text{Force coefficient along body x-axis} \\
  c_i & \quad \text{Slosh damping} \\
  c_m & \quad \text{Moment coefficient} \\
  s & \quad \text{Aerodynamic reference area in m}^2
\end{align*}
3. Introduction

Sloshing of liquid fuel in space launch vehicles is considered a problem and need to be addressed in much detail, especially on large space vehicle\(^1\). Launch vehicles and rockets usually contain significant amounts of liquid propellant; this has led to problems of instability arising from interaction between the propellant dynamics and the vehicle structural dynamics\(^2\). The slosh is developed in the fuel tank due to the dynamics of Launch Vehicle rigid members and this slosh forces will tend to change the actual dynamics of the system causing instability. The interaction between the structural members of the launch vehicle and sloshing part is considered to be a feedback type closed loop system [Fig. 1]. The problem is more acute in case of launch vehicles and rockets in which the slosh mass may form a significant fraction of the total vehicle mass, and where aerodynamic forces acting on the vehicle during atmospheric flight are relevant, but also, to a lesser extent, in aircraft\(^3\). Interaction between the slosh dynamics and the vehicle motion in flight may lead to periods of instability where self-excited oscillations first grow and then damp out on their own. A similar phenomenon, called engine-coupled pogo oscillation, is known to occur in liquid rockets\(^2\). Hence it is required to analysis the dynamic behavior of the vehicle system and to study the effect of
slosh on the system. With the computational estimation the slosh dynamic, the launch vehicle can be controlled to overcome this disturbance of slosh, so that a stable flight trajectory can be made highly possible. Here, the dynamic behavior of the launch vehicle system with and without slosh is discussed with three models

Figure 1: Schematics representation of vehicle-slosh interaction

and further, the control law will be synthesized to compensate the effect of slosh on the launch vehicle stability.
**Model 1**

**Simple configuration of launch vehicle system.**

Here in this model, the systems are developed as independent bodies having their own characteristic dynamics; behaves as rigid multi body system, when the independent system are coupled together [Fig. 2].

Planar motions of a vehicle in atmospheric flight with three degrees of freedom, two translational and one rotational, are considered. A single slosh mode is considered and is modeled as a non-linear simple pendulum oscillating in the same plane as that of the vehicle motion. The four-degree-of-freedom equations

![Figure 2: Simple configuration of the space launch vehicle system.](image-url)
of motion for the two-body system, vehicle and pendulum, are developed using the method of Lagrangian dynamics\textsuperscript{4}.

**Equation of motion:**

The co-ordinates of the vehicle center of mass with respect to some inertial frame of reference be \(x\) and \(z\). To obtain the equation of motion of the entire multi body system by Lagrangian- Euler method, it is required to generalize the co-ordinate system. Here \(X_p\) and \(Z_p\) represent the pendulum co-ordinates, the completely generalized co-ordinates system is given in the figure below.

![Figure 3: Generalised co-ordinate system of the pendulum.](image)

Then, the co-ordinates of the pendulum mass, \(m\) [Fig. 3]; will be

\[
\begin{align*}
x_p &= x + b \cos \theta - l \cos(\theta + \phi) \\
z_p &= z - b \sin \theta + l \sin(\theta + \phi)
\end{align*}
\]
The potential energy of the system, $V$; and the kinetic energy, $T$; are then given as

Potential energy $V$,

$$V = m_0gx + mgx_p + \frac{1}{2}k\theta^2$$

Kinetic energy $T$,

$$T = \frac{1}{2}m_0(x^2 + \dot{z}^2) + \frac{1}{2}m_0R^2\dot{\theta}^2 + \frac{1}{2}m(\dot{x}_p^2 + \dot{z}_p^2)$$

Here,

$g$ is the acceleration due to gravity and is assumed to be constant.

Lagrangian form $[L=T-V]$ of the system in given as,

$$L = \frac{1}{2}m_0(x^2 + \dot{z}^2) + \frac{1}{2}m_0R^2\dot{\theta}^2 + \frac{1}{2}ml(z^2 + b\dot{\theta}^2 \cos \theta + l \cos(\theta + \phi)(\dot{\theta} + \dot{\phi}))^2$$
$$+ (\ddot{x} - b\ddot{\theta} \sin \theta + l \sin(\theta + \phi)(\dot{\theta} + \dot{\phi})) - m_0gx$$
$$-mg(x + b \cos \theta - l \cos(\theta + \phi)) - \frac{1}{2}k\theta^2$$

Then, the Euler–Lagrange equation is

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$$

Where $i=1, 2, 3......$

The equation of motion of the system can be derived along the DOF as stated in Ref. 4.

**Equation of motion along Z-axis**

$$(m + m_0)\ddot{z} + m\ddot{\theta}(-b \cos \theta + l \cos(\theta + \phi)) + ml\ddot{\phi} \cos(\theta + \phi)$$
$$+mb\dot{\theta}^2 \sin \theta - ml(\dot{\theta} + \dot{\phi})^2 \sin(\theta + \phi) = F_z - C_\ddot{z}$$

Where, $F_x$ – Control force along in Z-axis direction.

$C_x$ – Coefficient of damping in Z-axis direction.
Equation of motion along X-axis

\[(m + m_\omega)\ddot{x} + m\ddot{\phi}(-b \sin \theta + l \sin(\theta + \phi)) + ml\dot{\phi}\sin(\theta + \phi) + mb\ddot{\theta}^2 \cos \theta + ml(\dot{\theta} + \dot{\phi})^2 \cos(\theta + \phi) + m_\omega g + mg = F_x - C_x \ddot{x}\]

Where

- \(F_x\): Control force along in X-axis direction.
- \(C_x\): Coefficient of damping in X-axis direction.

Pitching motion

\[m_\omega R^2 \ddot{\theta} + m\ddot{z}(-b \cos \theta + l \cos(\theta + \phi)) + m\ddot{\phi}(-b \sin \theta + l \sin(\theta + \phi)) + m\ddot{\theta}(b^2 + l^2 - 2bl \cos \phi) + ml\ddot{\phi} \sin \phi + mgb \sin \theta + mgl \sin(\theta + \phi) + ml\dot{\phi}\sin \phi + k\theta = M - C_\Theta \dot{\theta} - C_\phi \dot{\phi}\]

Where

- \(M\): Moment of LV about its center of mass
- \(C_\Theta\): Damping coefficient
- \(C_\phi\): Slosh damping coefficient

Sloshing motion

\[ml^2 \ddot{\phi} + mlz \cos(\theta + \phi) + ml\ddot{x} \sin(\theta + \phi) + ml\ddot{\theta}(l - b \cos \phi) + mlb \ddot{\phi}^2 \sin \phi + mgl \sin(\theta + \phi) = -C_\phi \dot{\phi}\]

Where

- \(C_\phi\): Slosh damping coefficient

The above derived equation of motion of the system is numerically instable due to singularity and numerical analysis can’t be performed unless the parameters are non-dimension.
Non-dimensional equation of motion

Displacement along x, $hx = \frac{x}{l}$

$hz = \frac{z}{l}$

Where,

$l$ is the length of pendulum

Force along $x$, $\bar{F}_x = \frac{F_x}{ml}$

Force along $z$, $\bar{F}_z = \frac{F_z}{ml}$

Moment, $\bar{M} = \frac{M}{ml^2}$

Damping coefficients,

$\bar{C}_x = \frac{C_x}{m_0}$  $\bar{C}_z = \frac{C_z}{m_0}$  $\bar{C}_\theta = \frac{C_\theta}{ml^2}$  $\bar{C}_\phi = \frac{C_\phi}{ml^2}$

Slosh mass fraction, $\epsilon = \frac{m}{m + m_0}$

Non-dimensional acceleration due to gravity, $\lambda = \frac{g}{l}$

Non-dimensional pendulum hinge point location, $\beta = \frac{b}{l}$

Non-dimensional pitch stiffness, $\bar{k} = \frac{k}{ml^2}$
Non-dimensional radius of gyration, $\rho = \frac{R}{l}$

**Non-dimensional equation of motion along Z-axis**

\[
\ddot{h}_z = \varepsilon(\dot{\theta} + \dot{\phi})^2 \sin(\theta + \phi) + \bar{C}_z(1-\varepsilon)\dot{h}_z - \bar{F}_z(1-\varepsilon) \\
-\varepsilon\dot{\theta}(\cos(\theta + \phi) - \beta \cos \theta) - \varepsilon\dot{\phi} \cos(\theta + \phi) \\
-\varepsilon\beta \ddot{\theta}^2 \sin \theta
\]

**Non-dimensional equation of motion along X-axis**

\[
\ddot{h}_x = \varepsilon\beta \ddot{\theta}^2 \cos \theta - \bar{C}_x(1-\varepsilon)\dot{h}_x + \bar{F}_x(1-\varepsilon) \\
-\varepsilon\dot{\theta}(\sin(\theta + \phi) - \beta \sin \theta) - \varepsilon\dot{\phi} \sin(\theta + \phi) \\
-\varepsilon(\dot{\theta} + \dot{\phi})^2 \sin(\theta + \phi) - \lambda
\]

**Non-dimensional pitching motion**

\[
\ddot{\theta} = \frac{1}{(\rho^2(1-\varepsilon) + \varepsilon\beta^2 + \varepsilon - 2\varepsilon\beta \cos \phi)[\varepsilon\lambda \beta \sin \theta} \\
-\bar{C}_\theta(1-\varepsilon) \dot{\theta} - \varepsilon\bar{C}_\phi \dot{\phi} + \bar{M}(1-\varepsilon) - \varepsilon \ddot{\phi}(1-\beta \cos \phi) \\
-\varepsilon \ddot{\phi}(\cos(\theta + \phi) - \beta \cos \theta) - \varepsilon \ddot{h}_z(\sin(\theta + \phi) - \beta \sin \theta) \\
-\varepsilon \beta \ddot{\phi}^2 \sin \phi + 2\varepsilon \beta \ddot{\theta} \sin \phi - \varepsilon \lambda \beta \sin \theta - \varepsilon \lambda \sin(\theta + \phi) \\
-\bar{k}(1-\varepsilon) \dot{\theta}
\]

**Non-dimensional sloshing motion**

\[
\ddot{\phi} = \beta \ddot{\theta}^2 \sin \phi - \bar{C}_\phi \phi - \dot{\bar{C}}_\phi(1-\beta \cos \phi) - \bar{h}_z \cos(\theta + \phi) \\
-\ddot{h}_z \sin(\theta + \phi) - \lambda \sin(\theta + \phi)
\]

The dynamic analysis of the launch vehicle system with the liquid sloshing is carried on the above derived non-dimensional equation of motion.
Matlab/Simulink plant model

Simulation Parameters:

l=1.1; Length of equivalent pendulum in m  
b=3.63; Distance of pendulum hinge point from c.g. in m  
g=9.8; acceleration due to gravity in m/s^2  
cz=0.1; Force coefficient along body z-axis  
cx=0.1; Force coefficient along body x-axis  
ci=0.003; slosh damping  
cm=0.1; Moment coefficient  
Mass of LV with fuel mass, mo=182610 kg  
Mass of fuel causing slosh, m=78261 kg  
Moment of inertia-pitching, M=22252820 kg-m^2  
R=11.03 m
Dynamic response of the simple system

The system is simulated for a time period of 100 seconds. The model is validated for stability, fixing the values of $M=0$; $F_x$ and $F_z$ as zero. It is inferred from the above plot, that slosh angle obtained as a function of the system dynamics. The resultant effect of the slosh perturbs the vehicle leading to the pitching motion. The simulation is repeated for various control inputs along $F_x$ and $F_z$, damping coefficient and for various moments, $M>0$. When $e$ is non-zero, the sloshing motion exerts slosh forces on the vehicle in the $x$ and $z$ directions (but no moment) at the hinge point. The slosh forces then create a moment about the rigid vehicle center of mass depending on the distance $b$ (or $b$ in non-dimensional form) of the pendulum hinge point from the center of mass. Thus, when $B=0$; the sloshing motion does not contribute any moment to the vehicle pitch dynamics.

Figure 4: plant model of simple launch vehicle configuration
The order of the above system is further reduced from 4 DOF to 3 DOF to obtain a transfer function on a control perspective.

**Reduced order model**

Defining small perturbations about the steady state, the linearized equations can be derived as follows

Along z-axis:

\[
\delta \ddot{h}_z + \epsilon \delta \ddot{\theta}(1 - \beta) + \epsilon \delta \ddot{\phi} = -\bar{C}_z (1 - \epsilon) \delta \dot{h}_z + \delta \bar{F}_z (1 - \epsilon)
\]

Along x-axis:

\[
\delta \ddot{h}_x = -\bar{C}_x (1 - \epsilon) \delta \dot{h}_x + \delta \bar{F}_x (1 - \epsilon)
\]

Pitch:

\[
\delta \ddot{\theta}(\rho^2 (1 - \epsilon) + \epsilon (1 - \beta)^2) + \epsilon \delta \ddot{\phi}(1 - \beta) + \epsilon \delta \ddot{h}_z (1 - \beta) + \epsilon \lambda (1 - \beta) \delta \theta
\]

\[+ \epsilon \lambda \delta \phi + \bar{k} (1 - \epsilon) \delta \theta = -\bar{C}_\phi (1 - \epsilon) \delta \dot{\theta} - \epsilon \bar{C}_\phi \delta \phi + \delta \bar{M} (1 - \epsilon)
\]

Slosh:

\[
\delta \ddot{\phi} + \delta \ddot{\theta}(1 - \beta) + \delta \ddot{h}_z + \lambda (\delta \theta + \delta \phi) = -\bar{C}_\phi \delta \phi
\]

**Figure 5: reduced order model**
From the above Simulink model, it is clearly seen that order of the system is reduced due to the negligence of force Fx along X-axis. Thus, it can be safely concluded that the x-dynamics neither participates in nor contributes to the instability phenomenon observed in the previous section. It therefore makes sense to ignore the x-dynamics equation of motion along X-axis and consider a reduced-order three-degree-of-freedom system.

This simple configuration model gives the dynamics of the vehicle system and the effect of slosh on it, but the external disturbance affecting the vehicle dynamics and the control thrust with gimbals deflection as seen in the actual launch vehicle are not addressed in this model, which concomitants the development of next model.

**Model 2**

**Coupled configuration with control input and aerodynamic drag**

Modified system with aerodynamic force as disturbance and gimbal deflection as the control input parameter.

The previously developed simple system is modified into an actual system by introducing aerodynamic drag force as the external disturbance and the control force Fx and Fy in the simple system is updated to a control input Tc (control thrust) having gimbals position with reference to the body axis of LV (Y). Aerodynamic reference area S, dynamic pressure q and air density are incorporated into the system equation and for simplicity of the system, the trim condition is assumed for the space flight. Aerodynamics is assumed to be linear and slosh is modeled using a simple pendulum model. Modeling of the coupled system based on the motion is allowed to perform only planar X-Z plane, which is an earth fixed inertial reference frame. The slosh-vehicle system in planar atmospheric flight is modeled as a two-body problem consisting of a rigid vehicle and a simple pendulum, representing the first lateral slosh mode in a tank, hinged at a point located off the vehicle axis, b. The equations of motion are derived using a Lagrangian formulation as it is easier to obtain an expression for the slosh force in the same way as done in the previous model 1.
The equation of motion is given by,

Along $Z$

$$(m + m_o) \ddot{z} + m \ddot{\theta} (-b \cos \theta + l \cos(\theta + \phi)) + ml \ddot{\phi} \cos(\theta + \phi) + mb \dot{\theta}^2 \sin \theta - ml (\dot{\theta} + \dot{\phi})^2 \sin(\theta + \phi) = F_{zc} + F_{ZA}$$

Figure 2: configuration of the space launch vehicle system with thrust $T_c$ control input.
Along $X$

\[(m + m_o)\ddot{x} + m\ddot{\theta}(-b \sin \theta + l \sin(\theta + \phi)) + ml\ddot{\phi} \sin(\theta + \phi)\]
\[-mb\ddot{\theta}^2 \cos \theta + ml(\dot{\theta} + \dot{\phi})^2 \cos(\theta + \phi) + (m + m_o)g = F_{xc} + F_{xA}\]

Along pitch

\[m_oR^2\ddot{\theta} + m\ddot{z}(-b \cos \theta + l \cos(\theta + \phi)) + m\ddot{x}(-b \sin \theta + l \sin(\theta + \phi))\]
\[+m\dot{\theta}\dot{z}(b^2 + l^2 - 2bl \cos \phi) + m\dot{\phi}(l^2 - bl \cos \phi) + 2mb\dot{\theta}\dot{\phi}\sin \phi\]
\[-mgb \sin \theta + mgl \sin(\theta + \phi) + mbl\dot{\phi}^2 \sin \phi = M_c + M_A - C_\phi \dot{\phi}\]

Along slosh

\[ml^2\ddot{\phi} + ml\ddot{z} \cos(\theta + \phi) + ml\ddot{x} \sin(\theta + \phi) + ml\dot{\theta}(l - b \cos \phi)\]
\[-mbl\dot{\theta}^2 \sin \phi + mgl \sin(\theta + \phi) = -C_\phi \dot{\phi}\]

Where,

- $F_{zc}$ and $F_{xc}$ are the control force
- $F_{za}$ and $F_{xa}$ are the aerodynamic force
- $M_A$ is the aerodynamic moment of the LV

Let $V_x$ and $V_z$ be the components of the velocity of the LV’s center of mass along $X_B$ and $Z_B$ respectively.

Then,

\[\dot{z} = v_z \cos \theta - v_x \sin \theta\]
\[\dot{x} = v_x \cos \theta + v_z \sin \theta\]
Defining,

$Z_c$ and $x_c$ are the control forces along $z_B$ and $z_B$ respectively
$M_c$ is the control moment
$Z_A$ and $x_A$ are the aerodynamic forces in $z_B$ and $x_B$ direction respectively

$$
Z_c = T \sin\psi = F_{zc} \cos\theta + F_{xc} \sin\theta \\
x_c = T \cos\psi = F_{xc} \cos\theta - F_{zc} \sin\theta \\
\mu_c = Tl_o \sin\psi
$$

Also,

$$
Z_A = C_z qS = F_{za} \cos\theta + F_{xa} \sin\theta \\
x_A = C_x qS = F_{xa} \cos\theta - F_{za} \sin\theta \\
\mu_A = C_m qS
$$

Substituting the above equation in the equation of motion for the coupled slosh-rocket system in terms of body quantities.

**Along Z**

$$(m + m_o)(\dot{v}_z + v_z \dot{\theta}) + ml \ddot{\theta} \sin\phi + ml \dot{\phi} \sin\phi + ml \dot{\phi} \sin\phi + (m + m_o)g \sin\theta - ml(\dot{\theta} + \dot{\phi})^2 \sin(\theta + \phi) = T \sin\psi + C_z qS$$

**Along X**

$$(m + m_o)(\dot{v}_x + v_z \dot{\theta}) + ml \ddot{\theta} \sin\phi + ml \dot{\phi} \sin\phi + ml \dot{\phi} \sin\phi - m b \dot{\theta}^2 + ml(\dot{\theta} + \dot{\phi})^2 \cos\theta + (m + m_o)g \cos\theta = T \cos\psi + C_x qS$$

**Along pitch**

$$m_o R^2 \ddot{\theta} + m_l^2 (\dot{v}_z + v_z \dot{\theta})(-b \cos\theta + l \cos\phi) + m(\dot{v}_z + v_z \dot{\theta}) l \sin\phi + ml \dot{\theta} (b^2 + l^2 - 2bl \cos\phi) + ml \dot{\phi} (l^2 - bl \cos\phi) + 2ml b \dot{\theta} \dot{\phi} \sin\phi - mg b \sin\theta + mgl \sin(\theta + \phi) + mbl \dot{\phi}^2 \sin\phi = Tl_o \sin\psi + C_m qS - C_{\phi} \dot{\phi}$$
Along slosh

\[ ml^2 \ddot{\phi} + m(\dot{v}_z - v_x \dot{\theta})l \cos \phi + ml(\dot{v}_x + v_z \dot{\theta}) \sin \phi + ml \dot{\theta}(l - b \cos \phi) \]

\[-ml\ddot{\theta}^2 \sin \phi + mgl \sin(\theta + \phi) = -C_{\phi}\dot{\phi} \]

Aerodynamic forces and moments have been written in term of non-dimensional co-efficient Cz, Cx and Cm.

S- Aerodynamic reference area

c- Reference length

q- Dynamic pressure

\[ q = \frac{1}{2} \rho (v_x^2 + v_z^2) \]

Where

ρ- Atmospherics air density

Figure 4: Simulink plant model of the LV-Slosh system
Based on the actual equation of motion, the Simulink plant model is developed. The control input is given by the gimbal position with reference to the LV main body axis about the c.g. This system is studied for the dynamic response under the trim condition of space flight. The simulation parameters are same as that of previous case, model 1.

As pitch angle $\theta$, rate of pitch $d\theta/dt$, slosh angle $\phi$ and rate of slosh $d\phi/dt$ are output of interest, a plot is made to compare the interested output variable as shown in the figure 5. This shows the effect of aerodynamic force on the launch vehicle system.

As discussed in the previous case, non-dimensional equation of motion gives more towards the control perspective.
Non-dimensionalised equation

Non-dimensional parameters are

\[ h_z = \frac{v_z}{l} \quad h_x = \frac{v_x}{l} \quad \bar{T} = \frac{T}{m_o l} \quad \bar{\mu} = \frac{q_s}{m_o l} \]

System of equation (non-dimensional)

Along z-axis,

\[ \dot{h}_z = h_x \dot{\theta} - \varepsilon \ddot{\phi} (\cos \phi - \beta) - \varepsilon \dot{\phi} \cos \phi + \varepsilon (\theta + \phi)^2 - \lambda \sin \theta + \bar{T} (1 - \varepsilon) \sin \psi + C_z \bar{\mu} (1 - \varepsilon) \]

Along x-axis,

\[ \dot{h}_x = -h_z \dot{\theta} - \varepsilon \ddot{\phi} \sin \phi - \varepsilon \dot{\phi} \sin \phi + \varepsilon \beta \theta^2 - \varepsilon (\theta + \phi)^2 \cos \phi - \lambda \cos \theta + \bar{T} (1 - \varepsilon) \cos \psi + C_x \bar{\mu} (1 - \varepsilon) \]

Along pitching axis,

\[ \ddot{\phi} = \frac{1}{\rho^2 (1 - \varepsilon)} [ -\varepsilon (1 + \beta^2 - 2 \beta \cos \phi) \dot{\phi} - \varepsilon (\dot{h}_z - h_x \dot{\theta}) (\cos \phi - \beta) - \varepsilon (h_x + h_z \dot{\theta}) \sin \phi - \varepsilon \ddot{\phi} (1 - \beta \cos \phi) - 2 \varepsilon \beta \dot{\phi} \sin \phi + \varepsilon \lambda \beta \sin \theta - \varepsilon \beta \dot{\phi} \sin \phi + \bar{T} (1 - \varepsilon) l_o \sin \psi + C_m \bar{\mu} (1 - \varepsilon) - \varepsilon \bar{C} \phi \dot{\phi} ] \]

Along slosh axis,

\[ \ddot{\phi} = - (\dot{h}_z - h_x \dot{\theta}) \cos \phi - (\dot{h}_x + h_z \dot{\theta}) \sin \phi - \ddot{\phi} (1 - \beta \cos \phi) + \beta \dot{\theta}^2 \sin \phi - \lambda \sin (\theta + \phi) - \bar{C} \phi \dot{\phi} \]
The above system is simulated for the extreme flight condition allowing the high rate of pitching due to the aerodynamic and sloshing effect. If a control law being developed for such a scenario to surpass the effect of slosh.

Figure 6: Simulink model of model 2 with KE validation (non-dimensional system)
Model 3:

GSLV model

ISRO had already developed the dynamic model of GSLV system and it is extensively studied in an M.Tech thesis\textsuperscript{5}.

Conclusion

Model 2, which is intentionally developed on control perspective. A controller based on sliding mode control law can be easily implemented on the model 2.
Reference:


