# Enticement to College Mathematics via Primary Historical Sources

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## Course Overview

The lower-division honors course *Spirit and Evolution of Mathematics* is based entirely on the philosophy of direct study of primary historical sources [1,2,3,7,9]. The main focus is on learning important and interesting mathematics, but with history as a constant natural companion, so that students learn much history in the context of learning specific mathematics. Each offering of the course is based on one or more sequences of (translated) primary sources, each sequence designed for studying one topic in depth. The intention is for students to experience as closely to firsthand as possible how human understanding of a great mathematical question or problem evolved over many centuries. Primary sources can naturally raise deep questions, and, through often long periods of historical turmoil, lead to deep understanding. Studying mathematics via this process avoids the “cart before the horse’’ method of most modern mathematics textbooks, which provide answers to questions that haven’t been asked.

“Study the masters!” is the course’s spirit, to examine great mathematical texts from a breadth of times and cultures, by immersing ourselves in the very words of the masters who first discovered new ideas. We see these ideas develop into modern branches of mathematics by studying selected sequences of primary sources. At the same time we consciously aim at developing an appreciation for and facility with methods of rigorous proof and modern mathematical thinking.

*Spirit and Evolution of Mathematics* began in 1989 as an honors option for meeting the lower-division general education mathematics requirement at New Mexico State University, and has been offered almost every year since then [4]. The prerequisite is “a good high school mathematics background”, in reality the same as for entry into precalculus, and no specific background topics are assumed. However, students are generally expected to have honors eligibility, which in practice means the ability to learn more independently and to contribute actively and innovatively to class discussion in a course with enrollment restricted to 20. At other institutions our materials have also been used for seminar courses at the upper level, or for individual topic study courses.

Students in the course hail from all possible majors, or are undeclared. In addition to general education, the course also attracts students to major in mathematics. Some of our very best mathematics majors have begun in this course, never imagining they would be attracted to major in mathematics, until studying primary sources opened their eyes and excited them to choose mathematics.

## Course Design

Our resource for the course is a collection of carefully chosen historical texts, ideally with generous annotation, commentary, and exercises. The goal is to study the original proofs of results in these texts, in the words of the discoverers, in order to understand the most authentic possible picture of the evolution of major branches of mathematics during a span of over two thousand years. It is exciting and illuminating to read original works in which great mathematical ideas were first revealed.

At home and in class we read, discuss, and interpret the theorems and their proofs, with students writing their thoughts and questions about these works, and we discuss how the various sources tie together in the development of important ideas. Students also solve challenging mathematical problem exercises directly related to the primary source texts.

Different instructors use different pedagogies in the classroom. Personally, I use a non-lecture three-part assignment method applied in almost all my teaching, explained in detail in [8], and in a handout for students (see Appendix C on homework guidelines). For each topic, students are first expected to read new material in advance of class, and to write questions about their mathematical reading for me to read before class. Second, students are expected to prepare mathematical work for class based on their reading, usually by attempting solutions of pre-assigned exercises. These two pre-class parts are graded based only on effort and preparation for class, very quick to assess with a plus, check, or minus.

In the classroom we first discuss their reading questions as a group, along with questions I inject. Class discussions are often challenging and fun for all, because the primary sources provide fabulous grist for deep and wide-ranging considerations. Part of the richness of studying primary sources is that today they often raise as many questions as they answer. The majority of classroom time is then spent with students working in informal groups on the previously assigned problems, interspersed with impromptu whole-class discussions or student board presentations suggested by me as common questions and interesting approaches arise.

The third part of each topic’s homework assignment consists of completing post-class homework on the topic, primarily a very few challenging exercises not worked on in class. Students are always encouraged to discuss their ideas with others, and then expected to finish and write up their polished post-class homework entirely on their own, in their own words, to hand in for me to read and mark carefully, and possibly to request rewriting for improvement. This post-class homework part ultimately receives a single letter grade for quality, one for each class day.

The course grade is based on a final holistic evaluation of student work: roughly one half on regular assignments (i.e., student writings on the original sources, and related mathematical assignments); roughly one quarter on class participation; and the remaining quarter on a term paper and a brief oral presentation of it, as described below. There are no exams. The course overview handout for students is in Appendix B.

The course has a very flexible timetable (hence there is no course outline appended), often influenced by what explorations happen in the classroom based on student response and activities, according to the classroom methods described above. Whether we engage one topic chapter or two from the textbook during a term varies.

On the first day I introduce and discuss the nature and expectations of the course, we dive into some mathematics, and I ask students to skim the entire five topic chapter materials of the textbook as homework, to provide feedback, as explained earlier, at the beginning of second class period. At the beginning of the second day I select a topic on the spot, based on their feedback, and we begin right away with primary source material from that topic. Succeeding days always have reading/writing in advance, preparatory mathematical work on exercises, in class group and whole class work, and final homework exercises, as described earlier. By mid-term students begin work on individual term paper research, in addition to our regular classroom work, as described earlier. The very end of term is spent on short term paper presentations and discussion.

# Resources

When this course was first co-created and taught in 1989, we had little more than a handful of chosen primary sources on a few topics, some of which we had to translate ourselves, often with no annotation, context, or exercises. We assigned some essay readings to supplement the sources, created assignments as we went, and started writing annotation to tie it all together. And indeed this is how anyone can still develop their own materials; I enthusiastically recommend it. Guidance on the pedagogical principles, and on design of materials, can be found in [1,2,3,7,9]. Today primary sources are much more easily available, and in translation as well, than when we started. The reader may be pleasantly surprised that finding promising and appropriate primary sources for teaching on a given topic is not as hard as may be feared. The bibliography [10] provides a window to many historical sources for teaching. The recent sizeable source book [12] would be a good place to find many good sources on a variety of topics (see the review [11]).

Over the years we settled on and developed five topic sequences of primary sources, translated as necessary, on grand topics in the evolution of mathematics, added extensive annotation, contextual, historical, and mathematical commentary as a guide and overview of the big story, and numerous mathematical exercises for students. We also included copious references to the literature for deeper understanding by both teachers and students. These became the five chapters of the textbook *Mathematical Expeditions: Chronicles by the Explorers* [5]. Each chapter has an extensive Introduction, which tells a large story from the beginning, both mathematically and historically. As it proceeds, the Introduction points the reader to the subsequent chapter sections, which focus in sequence on sources by specific authors. Therefore we have students read the Introduction in tandem with work on individual chapter sections, going back and forth between the main story and the featured primary source sections.

Each one-semester offering of the course typically covers only one or two of the five independent chapters in depth. Students are sometimes asked to start by skimming the book as homework and rating (A-F) and ranking (1-5) the five chapters for interest, before I select which chapter we will study. The point of both rating (absolute) and ranking (relative) is to aim to choose a chapter that most students will find inspiring, and that they will feel was a preferred choice.

The chapter themes and authors of the primary sources we have used are:

- *Geometry: The Parallel Postulate*. The primary sources follow two millennia of the development of non-Euclidean geometry via excerpts from Euclid featuring his parallel postulate, Legendre's early nineteenth-century final attempts to prove the parallel postulate, Lobachevsky's almost simultaneous introduction of the brave new world of planar hyperbolic geometry, and Poincaré's disk model confirming its equal footing with Euclidean geometry.
- *Set Theory: Taming the Infinite*. Bolzano considers explicit mathematical paradoxes of the infinite, Cantor confidently opens the cornucopia of explicit different infinite cardinalities, and Zermelo axiomatizes set theory to give foundations to modern mathematics.
- *Analysis: Calculating Areas and Volumes*. Primary sources follow the development of calculus over more than two millennia. Archimedes calculates areas of parabolic sectors by the method of exhaustion, and reveals his technique of balancing indivisibles for discovering results before proving them by exhaustion. Cavalieri calculates areas of higher parabolas using geometric algebra and indivisibles. Leibniz proves the fundamental theorem of calculus with his infinitesimals, Cauchy rigorizes integration as limits of discrete summations, and Robinson resurrects infinitesimals in the twentieth century.
- *Number Theory: Fermat's Last Theorem*. Euclid classifies all Pythagorean triples, Euler proves Fermat's Last Theorem for exponent four, Germain provides the first general approach, proving many instances of Case I, and Kummer elucidates the beginnings of algebraic number theory.
- *Algebra: The Search for an Elusive Formula*. Primary sources follow the quest for formulaic solutions to polynomial equations. Euclid resolves quadratics with plane geometry, Cardano solves cubics with verbal geometric algebra, Lagrange explores the

fading promise for higher degrees, and Galois converts the problem to studying groups of permutations of roots and field extensions.

As illustrations, Appendix A introduces small excerpts from selected primary source material for each theme, along with connected sample exercises for students. The website [6] provides sample sections from each chapter.

# Assignments

Regular homework and related classroom work are the heart of the course. Assignments are largely mathematical in nature, based directly on the primary sources, since the course is first and foremost mathematics, set authentically in its history. Exercises often strengthen students' understanding of a primary source, and are sometimes open-ended. To give a diversity of flavors, Appendix A provides sample exercises from our five general themes above, each exercise preceded by a little context and a small excerpt from the relevant primary source.

For the term paper and brief oral presentation (see handout for students in Appendix D), the choice of topic is up to the student, subject to my approval, but should include a meaningful mathematical component (not mostly biographical) that they can genuinely understand and explain to others in a presentation. I do not suggest topics, so students must keep their eyes open for something along the way of interest. I emphasize that this is an opportunity to delve into something personally exciting or innovative. This responsibility is consciously placed on each student, to try to encourage them to take initiative. One obtains a great variety of quality in topics, and while not all are inspiring, some are truly fascinating.

Because a lot of independent judgement is being expected of lower division students in choosing a topic and writing a term paper, one needs to monitor and guide the process closely so that individual students don't end up way off the desired mark, creating disasters for them and the instructor. So students are asked to pick a term paper topic by mid-semester, and I help students in refining ideas for a topic. Each student is first asked to come up with two ideas for a topic, do a preliminary library search to see that adequate research materials are to be found there (required usually to be books, not just internet sources), and write a paragraph describing each topic to me, along with references to what was found in the library. After possible further refinement, often having them show me their resource books, I approve each topic. Students are sometimes required to show me their writing progress along the way, to help them complete an acceptable paper on time. We use the mandatory final exam period, as well as the last few class days as needed, for term paper presentations, with papers due several days before presentations.

Very occasionally a student will not start their term paper in time, despite my best efforts to guide them, and out of desperation submit something plagiarized; a student who requests to change their topic at the last minute is a possible indicator of this. One needs to be on the lookout for this quite rare occurrence, try to prevent it from happening, but be prepared if it does.

# Lessons Learned

This is a highly engaging course for students interested in mathematics, no matter what their major, and some students go on unexpectedly to study more mathematics after having their interest sparked. Many students absolutely love the primary sources, and especially those from the humanities take to them very enthusiastically.

It is important that enrolled students share a certain minimum level of ability with the mathematics, since otherwise they will not be able to cope with the challenge of the primary sources. Our current prerequisite is roughly a Mathematics ACT score of 25, and students allowed into the course without this level often do not succeed.

The class discussions are usually highly stimulating for both students and instructor, since grappling with primary sources invariably leads directly to deeper questions than a textbook usually does. But there is often great variation in how students choose to participate, if left to their own devices, so the instructor needs to be constantly aware of this, and act to guide the process. For instance, one needs to try to draw out the quieter students and gently reign in more outspoken ones, both in group work and in whole-class discussion.

Overall the greatest instructor challenge with this course is simply that the students are lower division, often entering freshmen, and the expectations of this course are frequently a big step up from what they have accomplished before. For students without the prerequisites it is usually too big a step.

# References

1. J. Barnett, J. Lodder, and D. Pengelley, The pedagogy of primary historical sources in mathematics: classroom practice meets theoretical frameworks, *Science & Education,* 23 (2014) 7-27.

2. J. Barnett, J. Lodder, and D. Pengelley, Teaching and learning mathematics from primary historical sources, *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 26 (2016) 1-18.

3. J. Barnett, J. Lodder, D. Pengelley, I. Pivkina, and D. Ranjan, Designing student projects for teaching and learning discrete mathematics and computer science via primary historical sources, in *Recent Developments in Introducing a Historical Dimension in Mathematics Education,* V. Katz and C. Tzanakis, eds., Mathematical Association of America, Washington, D.C, 2011, 189-201.

4. R. Laubenbacher and D. Pengelley, Great problems of mathematics: A course based on original sources, *American Mathematical Monthly,* 99 (1992) 313-317. Also at http://www.math.nmsu.edu/~history/.

5. R. Laubenbacher and D. Pengelley, *Mathematical Expeditions: Chronicles by the Explorers*, Springer-Verlag, New York, 1999. Excerpts and reviews at http://www.math.nmsu.edu/~history/).

6. R. Laubenbacher and D. Pengelley, *Teaching with Original Historical Sources in Mathematics,* a resource web site, http://www.math.nmsu.edu/~history/, 1999.

7. R. Laubenbacher, D. Pengelley, and M. Siddoway, Recovering motivation in mathematics: Teaching with original sources, *Undergraduate Mathematics Education Trends,* 6 No. 4 (September, 1994), parts of pages 1,7,13. Also at http://www.math.nmsu.edu/~history/.

8. David Pengelley, *Comments on Classroom Dynamics*, 2010, at http://www.math.nmsu.edu/~davidp/.

9. David Pengelley, Teaching with primary historical sources: Should it go mainstream? Can it?, in *Recent Developments in Introducing a Historical Dimension in Mathematics Education,* V. Katz and C. Tzanakis, eds., Mathematical Association of America, Washington, D.C., 2011, 1-8. Also at http://www.math.nmsu.edu/~davidp/.

10. David Pengelley, *Some Selected Resources for Using History in Teaching Mathematics*, 2011, http://www.math.nmsu.edu/~history/resources.html. This list includes numerous collections of primary sources, collected works, historically oriented mathematics books, articles on teaching using history, historically oriented teaching materials, reference works, specialized and general histories, periodicals, and web resources.

11. David Pengelley, book review of *Mathematics Emerging: A Sourcebook 1540-1900*, by Jacqueline Stedall, *Notices, American Mathematical Society,* 58 (2011) 815-819.

12. Jacqueline Stedall, *Mathematics Emerging: A Sourcebook 1540-1900*, Oxford University Press, Oxford, 2008.

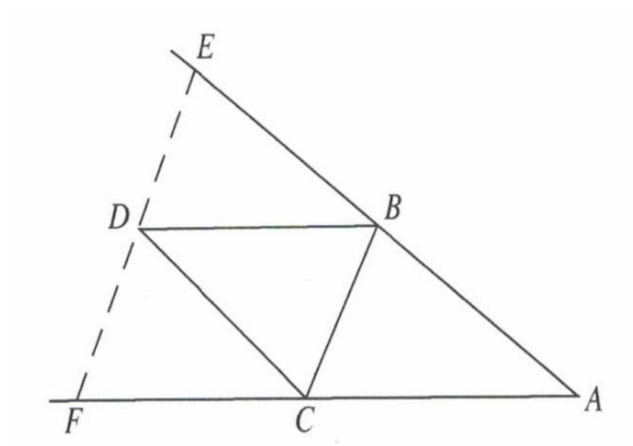
# Appendix A: Sample primary source materials and exercises

Here I describe a small primary source excerpt and related exercise for each major theme we have used in teaching the course.

- *Geometry: The Parallel Postulate*

Legendre was the last serious mathematician to attempt to prove Euclid's parallel postulate, while at essentially the same time Gauss, Lobachevsky, and Bolyai were developing the non-Euclidean hyperbolic geometry that negated it. From Legendre's many published attempts, students study an entire proof that the angle sum of a triangle must be two right angles (equivalent to the parallel postulate), and are challenged to find an unsupported assumption. Legendre's text contains the passage

*"Let A be the smallest of the angles in triangle ABC, on the opposite side BC make the angle BCD=ABC, and the angle CBD=ACB; the triangles BCD, ABC will be equal, by having an equal side BC adjacent to two corresponding equal angles [pr. 7]. Through the point D draw any straight line EF which meets the two extended sides of angle A in E and F."*




Students are first challenged, from amongst the entire proof, to ferret out that a subtlety in this passage is the key:

*Exercise: Before reading our commentary after Legendre's results, find and discuss the flaw in his proof of the parallel postulate.*

- *Set Theory: Taming the Infinite*

Students study Cantor's famous diagonal argument in his own words, including the passages

*"Namely, if m and w are any two distinct characters, we form a collection M of elements $E=(x_1,x_2, \dots ,x_\nu, \dots)$ which depends on infinitely many coordinates $x_1,x_2, \dots ,x_\nu, \dots$, each of which is either m or w. Let M be the set of all elements E.*

*…*

*I now claim that such a manifold M does not have the power of the series $1,2, \dots ,\nu, \dots$ .*

*…*

*This proof appears remarkable not only due to its great simplicity, but in particular for the reason that the principle employed in it can be directly extended to the general theorem, that the powers of well-defined point sets have no maximum, or what amounts to the same, that to every given point-set L can be associated another one M which has a higher power than L."*

Students are then challenged to extend Cantor's theorem to arbitrary cardinalities, and to construct infinitely many infinite cardinals.

*Exercise: Generalize Cantor's argument in the last source to prove that $2^{\mathfrak{m}}>\mathfrak{m}$ for any cardinal number $\mathfrak{m}$.*

*Exercise: List an infinite sequence of infinite cardinal numbers.*

- *Analysis: Calculating Areas and Volumes*

One of Archimedes' triumphs was his determination, without modern calculus, of the area of any segment cut from a parabola. Students study his entire proof by the Greek method of exhaustion. Archimedes states his result as

*"For it is here shown that every segment bounded by a straight line and a section of a right-angled cone [a parabola] is four-thirds of the triangle which has the same base and equal height with the segment."*

One exercise for students is to connect this with our modern calculational approach.

*Exercise: Archimedes proves that the area of a segment of a parabola is four-thirds that of a certain triangle. But this is of limited usefulness unless one can determine the dimensions of the triangle for a given segment, in order to compute its area. Show how this can always be done.*

- *Number Theory: Fermat's Last Theorem*

Sophie Germain was the first to provide general results towards proving Fermat's Last Theorem. Students study her unpublished manuscripts, including the main result she has been known for:

*"If p is an odd prime, and if there exists an auxiliary prime $\theta$ with the properties that p is not a p-th power modulo $\theta$, and the equation $r' \equiv r+1$ modulo $\theta$ cannot be satisfied for any two p-th power residues, then Case I of Fermat's Last Theorem is true for p."*

With Germain's Theorem, students can start proving special cases of Fermat's Last Theorem.

*Exercise: Use Germain's Theorem to show that there are no Case I solutions to the Fermat equation for exponent 7.*

- *Algebra: The Search for an Elusive Formula*

The search for the elusive roots of polynomials had many twists and turns. One was a poem divulged to Cardano by Tartaglia for solving a certain type of cubic equation:

*"When the cube and its things near*
*Add to a new number, discrete,*
*Determine two new numbers different*
*By that one; this feat*
*Will be kept as a rule*
*Their product always equal, the same,*
*To the cube of a third*
*Of the number of things named.*
*Then, generally speaking,*
*The remaining amount*
*Of the cube roots subtracted*
*Will be your desired count."*

An exercise for students is to decipher how this poem yields a modern looking formula.

*Exercise: Turn Tartaglia's poem into the formula in the Introduction.*

# Appendix B: Handout for students on course overview

**SPIRIT AND EVOLUTION OF MATHEMATICS**

Prerequisites: A good high school mathematics background: for instance, a Mathematics ACT score of 25 or better, or meet placement requirements for entry into precalculus, or consent of instructor.
Course Text: *Mathematical Expeditions: Chronicles by the Explorers*, by R. Laubenbacher and D. Pengelley (at the bookstore).

Study the masters! is the spirit of this course, in which we will study great mathematics from a breadth of times and cultures, by immersing ourselves in the very words of the masters who first discovered new ideas. We will see these ideas develop through time into modern branches of mathematics, by studying selected sequences of primary historical sources. Themes may include the emergence of non-Euclidean geometry, the concept of the infinite, the calculus, number theory, and the quest to solve algebraic equations. At the same time we shall aim at developing an appreciation for and facility with methods of rigorous proof and mathematical thinking.

Our primary objects of study will be a collection of historical mathematical texts with annotation and commentary. The goal is to study the original proofs of the theorems in these texts, in the words of the discoverers of the mathematics, in order to understand the most authentic possible picture of the evolution of major branches of mathematics during a span of over two thousand years. It is exciting and illuminating to read original works in which great mathematical ideas were first revealed. At home and in class we will read, discuss and interpret these theorems and their proofs, with you writing your thoughts and questions about these works, and we will discuss how the various sources tie together in the development of important ideas. I expect our class discussions to be both challenging and tremendous fun. Other written assignments will consist of proving related results, and solving other mathematical problems related to the texts.

You will have the opportunity to work both individually and collaboratively with others. I encourage you to discuss your ideas with others, and then I expect you to write up your homework entirely on your own, in your own words. As we examine the development of mathematical ideas in the original texts, we will also discuss their historical context and biographies of their creators.

I always expect to receive your homework when due, unless there are extenuating circumstances you discuss with me; speak with me if your homework must be late for a special reason. You should always hand in your homework when due even if it is incomplete. Late homework will receive only partial or no credit. On-time homework may sometimes, at my suggestion, be re-worked after I critique it, to bring it to perfection, due at the next class period after being returned by me. My goal is to help you perfect your work to your and my satisfaction.

I will expect active class participation from everyone, since this is one of the most valuable and enjoyable features possible in a small Honors class. Thus in particular I expect everyone to attend regularly. Please also be here so we can all start class on time, since late arrivals are particularly disruptive for a small class based primarily on discussion. Our class environment should be devoted entirely to our joint endeavors. In particular, I expect that we should never, ever be interrupted by a cell phone, even a buzz or vibration. Please take the responsibility to make absolutely sure that any cell phones remain completely turned off at all times in the classroom.

I will expect you to write a term paper and give a brief oral presentation on it. The choice of topic will be up to you, with my approval, but should include a meaningful mathematical component (in particular, it should *not* be mostly biographical). So keep your eyes open for something along the way of interest, since this is an opportunity to delve into something particularly exciting to you. I will discuss the timetable for the development of your paper in class, and will ask you to pick a topic by mid-semester. I will help you in refining your ideas for a topic. Please familiarize yourself with the university's policy regarding plagiarism and academic misconduct at www.nmsu.edu/~vpsa/SCOC/misconduct.html. I will discuss this further when you select a paper topic.

Your course grade will be based on a final holistic evaluation of your work as a whole: roughly one half on regular assignments (i.e., your writings on the original sources, and related mathematical assignments); roughly one quarter on class participation; and the remaining quarter on your term paper. There will be no in-class exams. However, we will use the mandatory final exam period for class activity, probably term paper presentations; please plan to be there. Please note that in Honors College courses, if you are taking the course on an S/U basis, a B- is required to produce an S.

**Regarding what I write on your papers**: My goal is to help you learn. If I write a note asking you a question, write back and bring my attention to it, or talk to me! If I circle something and put a question mark, I could not decipher or understand it. If I circle something with no comment, I am alerting you to check on a question of spelling, grammar, or meaning that needs correcting. If something needs further work, I will often expect you to redo it to perfect it, and give credit when it is corrected.

**Help**: Here are places to go for help.

a) I strongly encourage group work on homework, with individual write-ups. The only **rule** is that if you are to hand in your own individual assignment, then after you've talked all you want with sources of help, go home and write up your own assignment, *by yourself*, to hand in. Your paper should never read like anyone else's, since it should always be your own thoughts in your own words.
b) Also, I'm available, and want to help. I should be your primary source of help after those above. Try to prepare specific questions about things you are having trouble with. “I can't do this problem” is not very helpful to you or me, whereas “I started to do this, and then this happened, and I got stuck” gives us much more to work with. I can help you a lot in class, and am also available during my office hours, or by appointment.

# Appendix C: Handout for students on homework assignment guidelines

**Keep this sheet**

**Guidelines for all regular homework assignments**

Please put your name (and any nickname your prefer) on the first page, *staple* your pages together, and *do not* fold them. Use both sides of the paper if you wish, to save paper. Please *do not* write in light pencil. Please write clearly. Thank you.

Parts A, B, C of each homework are equally important.

**Part A: Advance preparation. Hand this in at the beginning of class, one class period before our class discussion and work on new reading. Reading responses (a), questions (b), reflection (c), and time spent (d):**

You do *not* need a new page for each part (a),(b),(c),(d).

a) Read assigned material. Reread as needed for complete understanding. Then write clear *responses* to assigned questions about the reading.

b) Write down some of your own explicit *questions* about your reading, ready to bring up in class. This may involve new or old concepts which are confusing to you, and connections to other ideas. You should also consider writing down what was well explained and interesting, what was confusing, what you had to reread but eventually understood.

c) Reflection: Write two or three sentences *reflecting* on the process of your work; this should only take a few minutes. Write about how things went with any assignment or reading done for class, and other course work. This should reflect both your ongoing personal feelings about the course as a whole and your interaction with the material at hand.

d) Write how much *time* you worked on part A.

**Part B: Warmup exercise preparation to present in class. This is due during class when we begin to discuss new material.** Work individually, and then with others in your group outside class time, on a few assigned easy warmup exercises on the new material we will discuss, based on your advance reading in Part A. Write up the solutions to these individually, to hand in in class. I will ask individuals and groups to present some of these to the class, to get us started discussing new material. Be sure to hand these in before leaving class.

Also always write how much time you worked on part B, and with whom.

**Part C: Main exercises. These will be assigned after class discussion and work on new material. They will normally be due next period.** Work individually and with others in your group on these. Also come to see me during office hours or at other appointment times about these. I am happy to help you. Then go home and write up your final solutions completely by yourself, without comparing with other people. The paper you hand in should be entirely your own writing, not the same as anyone else's.

# Appendix D: Handout for students on term paper guidelines

**Guidelines for term paper and presentation**

One quarter of your work for the course is a well written term paper on a mathematical topic of your choice, along with a brief class presentation.

- I will ask you to take the initiative in finding a term paper topic. First I will ask you to come up with two ideas for topics, do a preliminary library check that adequate research materials are likely to be found there, and write one paragraph describing each topic to me, along with references to what you found in the library. Use your imagination and interests in selecting topics!
- The principal requirement for a topic is that it should be primarily about mathematics in some form. If it is about a mathematician, then it should focus at least half on that person's mathematical work, and not more than half on biography. The other main requirement is that you should be able to discuss the mathematics in your paper with some genuine understanding of it. Writing a paper that lists mathematical results you have absolutely no understanding of is not fruitful.
- I will give you feedback on your tentative topics, and may ask you to seek further source material in the library to show me, to make sure a topic is appropriate. Then I will approve your selection of a topic.
  Please read http://lib.nmsu.edu/plagiarism.
  - I expect your paper to be well written. It should include a complete list of references. If you wish to quote directly from a source somewhere in your paper, instead of writing something in your own words, you should indicate the quotation clearly. Any consistent format for the paper and references is acceptable.
  - Your paper should be about 10 pages long.
  - You may handwrite your paper clearly. If you type it, please use 1½ or double spacing.
  - I may suggest that you show me a rough draft or outline before the final version of your paper.
  - When you give me your paper, keep a copy to prepare for your presentation to the class.