

# *Essential Mathematics for Geospatial Technicians*



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Engineering New Mexico  
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# Objectives

- Measurement units and unit conversions
- Pythagoras theorem
- Right triangle trigonometry
- Angles, Directions, Azimuths
- Zenith angles and slopes
- Coordinates – Forward and Inverse
- Circular curve geometry

# Sexagesimal Units

Sexagesimal (base 60) is a numeral system with sixty as its base. It originated with the ancient Sumerians in the 3rd millennium BC, it was passed down to the ancient Babylonians, and it is still used — in a modified form — for measuring time, angles, and geographic coordinates.

The number 60, a highly composite number, has twelve factors, namely {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}, of which 2, 3, and 5 are prime numbers. With so many factors, many fractions involving sexagesimal numbers are simplified. For example, one hour can be divided evenly into sections of 30 minutes, 20 minutes, 5 minutes, 12 minutes, 10 minutes, 6 minutes, 5 minutes, 4 minutes, 3 minutes, 2 minutes, and 1 minute. 60 is the smallest number that is divisible by every number from 1 to 6; that is, it is the lowest common multiple of 1, 2, 3, 4, 5, and 6.

# Unit Conversions

DMS to decimal:  $DD = \text{Deg} + \text{Min}/60 + \text{Sec}/3600$

Decimal to DD MM SS:

\*Strip off whole degrees

\*take fractional part \* 60 = take integer minutes

\*take fractional part \* 60 = Seconds and decimal seconds

Decimal degrees to radians:  $\text{Radians} = DD * (\pi/180)$

Radians to decimal degrees:  $DD = \text{rad} * (180/\pi)$

# Other units

## Units of Linear Measure

1 chain = 100 links

= 66 feet (U.S. Survey Foot)

= 4 poles, perches, rods

1 mile = 80 chains

= 5,280 feet (U.S. Survey Foot)

## Units of Area

1 acre = 10 square chains

= 43,560 square feet (U.S. Survey Foot)

1 square mile = 640 acres

# Other units

## Metric Conversions

U.S. Survey Foot

1 meter = 39.37 inches (exact)

1 U.S. Survey Foot = 0.3048006096... meter

1 link = 0.2011684023... meter

1 meter = 3.2808333333... U.S. Survey Foot

1 acre = 0.40468726099... hectare

International Foot (SI)

1 inch = 25.4 millimeters (exact)

1 SI Foot = 0.3048 meter (exact)

1 meter = 3.2808398950... SI Foot

# The concept of PI ( $\pi$ )

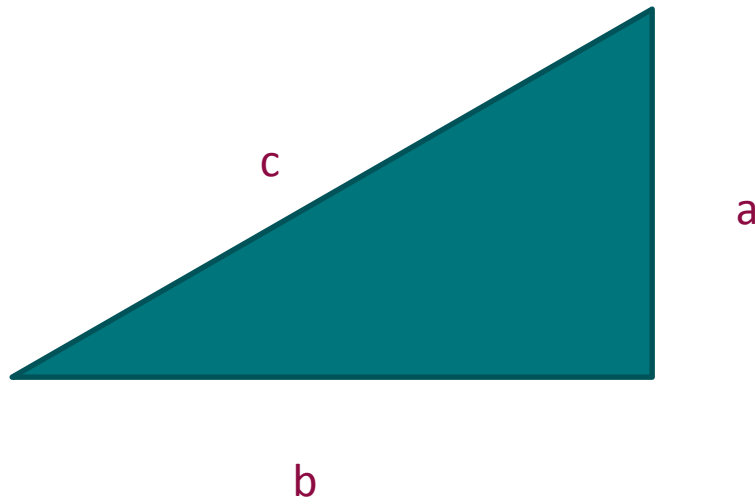
Pi is an irrational number that is the relationship between the circumference of a circle having a diameter of 1.

Pi, therefore is a required essential element when expressing an angle in radians.

One full revolution (360 degrees) =  $2 * \pi$

One semicircle (180 degrees) =  $\pi$

# Pythagoreans theorem



If two sides of a right triangle are known, the third side can be computed.

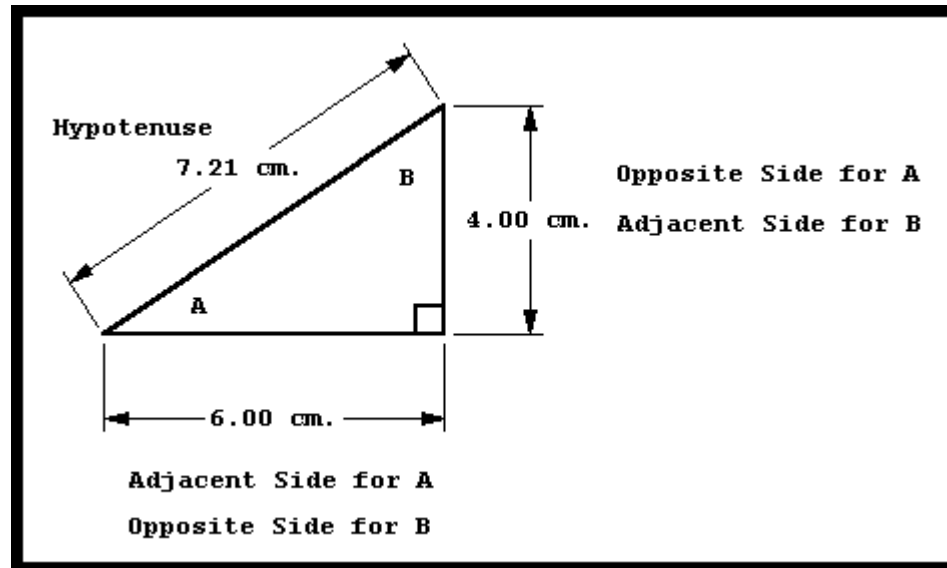
$$c^2 = a^2 + b^2$$

-or-

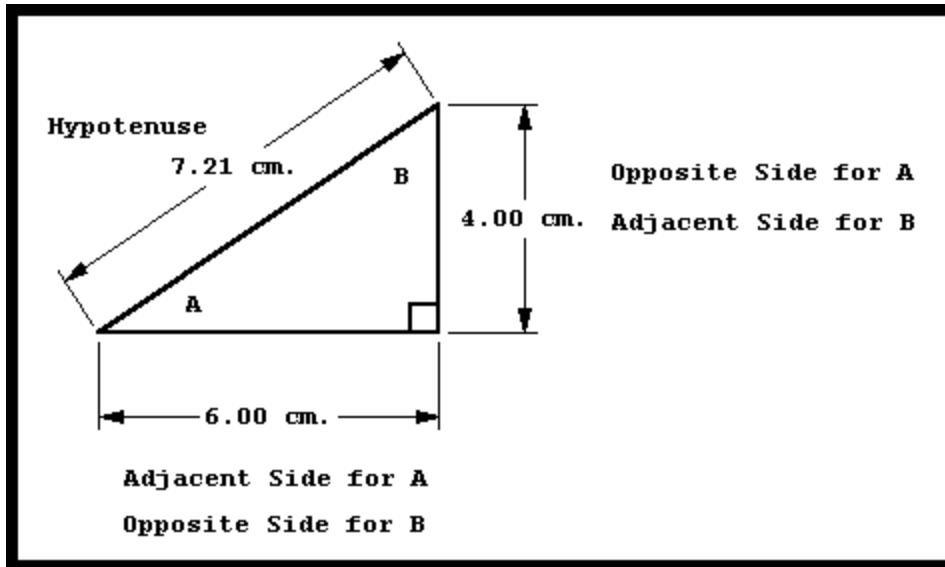
$$c = \sqrt{a^2 + b^2}$$



# Right triangle trig



# Right triangle trigonometry



## SOHCAHTOA

Sine A = Opposite/Hypotenuse

$$\text{Sine A} = (4.00/7.21)$$

$$\text{Angle A} = \text{ArcSine} (4.00/7.21)$$

$$\text{Angle A} = \text{Sin}^{-1} (4.00/7.21)$$

Cosine A = Adjacent/Hypotenuse

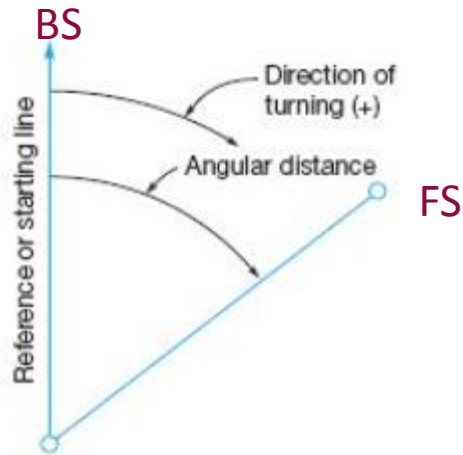
Tangent A = Opposite/Adjacent

Sine is typically abbreviated to SIN, Cosine abbreviates to COS, Tangent abbreviates to TAN

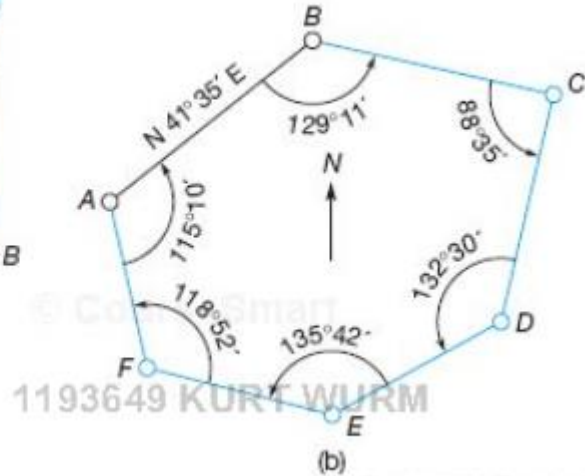
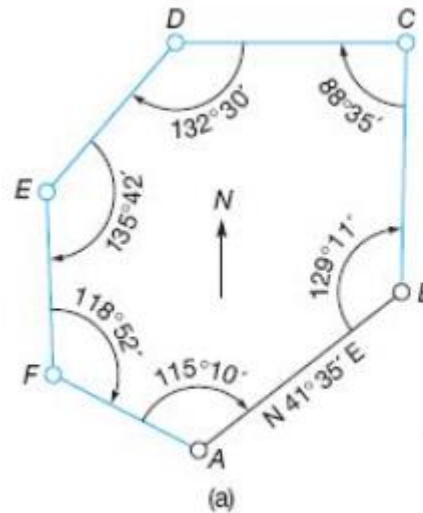
The inverse functions are often displayed as -1 power or use of the prefix "Arc"

example: Arcsin =  $\text{Sin}^{-1}$

# Angles and directions



Traverse with clockwise and counterclockwise angles



To avoid this confusion, it is recommended to always measure angles to the right. This is easily setup in modern total stations

# Angles and directions

- Due to the random error in our instruments, Angles should always be measured direct and reverse, a total of 4 pointings (or more for greater precision)
- The direction is read at the BS and the FS, these must be differenced to derive the angle between. (FS-BS)
- The direction is read in the reverse face, and reduced as above
- The two derived angles are then averaged. This effectively removes systematic error in the instrument.
  
- For efficiency, we often set the initial direction to the BS to Zero.
  
- BE CAREFUL, If an angle accumulates to greater than  $359^{\circ}59'59''$ , our instruments will go back to zero (thus you may have to add  $360^{\circ}$  to the direction displayed).

# Zenith angles

- Observing zenith angles is performed by observing the Z in both the direct and reverse positions and must also be averaged.
- The  $0^\circ$  position of the zenith is at a point directly above the instrument.
- At  $90^\circ$ , the instrument is pointing in a horizontal direction, in the reverse position, this is  $270^\circ$
- To average zenith angles:  $[(360 - \text{Reverse } Z) + \text{direct } Z]/2$
- Use care with units.

# Slope and horizontal distance

The EDM component of a total station operates on the line-of-sight. Therefore, all observed distances are along that line of sight.

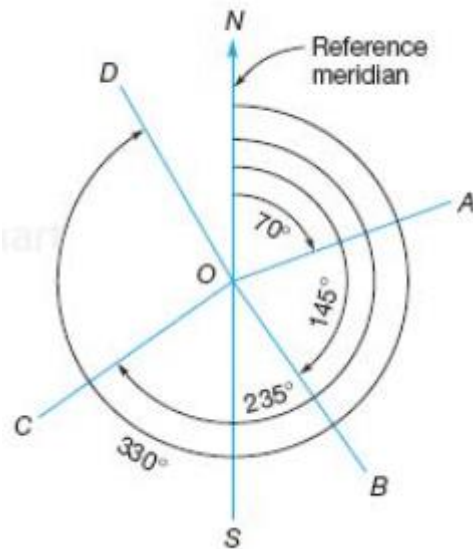
To compute the Horizontal Distance from an observed average of a set of slope distances and averaged Zenith angle:

	°	'	"	dd	meanZ	SD	HD	VD
Direct	90	11	35	90.19305556		1000.000		
Reverse	269	48	7	90.19805556				
					90.19555556		999.994	-3.413

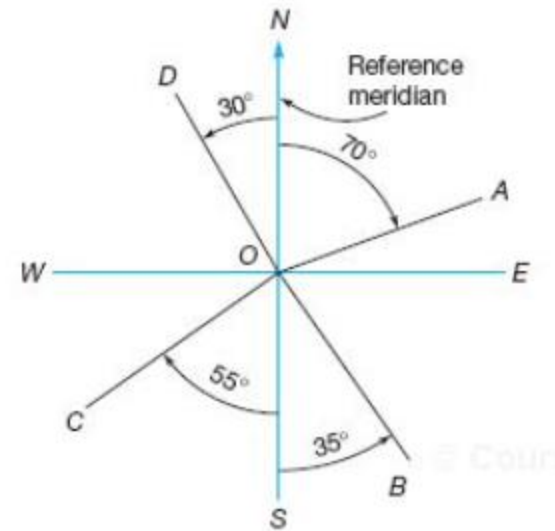
$$HD = SD * \sin(\text{meanZ})$$

$$VD = SD * \cos(\text{meanZ})$$

# Directions and Azimuths

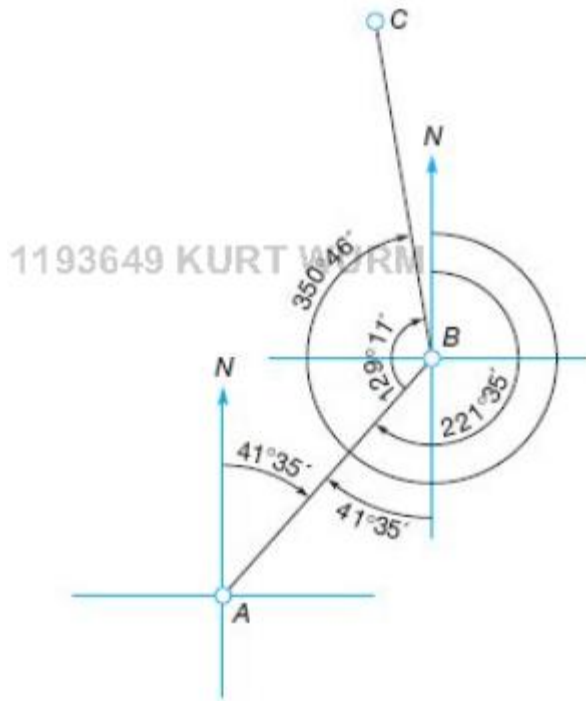


Azimuths: Clockwise from North



Bearings: East or West of North or South

# Computing Azimuths



The forward azimuth AB is  $41^\circ 35'$

To compute the forward azimuth BC:

a) add 180	$(221^\circ 35')$
b) add angle right	<u><math>+(129^\circ 11')</math></u>
	$350^\circ 46'$

Most calculators will require each of these angles to be expressed in decimal degrees. It is common practice to convert the answer back to DMS.



# Calculator modes of operation

- The default setting in most handheld calculators is DEGREES mode, meaning decimal degrees. Therefore, to take the SIN, COS, or TAN of an angle, it must first be converted to DD.ddddd format. (do not round off, try to use full precision)
- This also means that Arcsine, Arccosine, and Arctangent functions will return an angle in DD.ddddd format.
- Most computer software such as MS-EXCEL operate in RADIANS mode. Therefore, an angle in DMS must first be converted to DD, then to radians for trig functions to work properly.
- Angle arithmetic and typically be conveniently performed in decimal degrees. The caution about radians is concerning trig functions.

# Total Stations and Data Collectors

- For the purpose of ease of use, our modern data collectors often allow one to setup over a known station, enter its point ID or coordinates, same for the BS
- The total station is triggered at each of the pointings, software reduces the angles, and computes the forward azimuth.
- Distances are always measured on the slope, by automatically reducing the zenith angles also, the computation of Horizontal Distance is automated.
- The azimuth and HD are used to compute the coordinates of the FS station.

# Forward Solution

With horizontal angles reduced, forward azimuths computed, Zenith angles reduced and horizontal distances computed, we can now begin to compute coordinates!

In mathematics, coordinates are typically referred to as X and Y, in surveying, we often refer to these as Northings (y coordinates) and Eastings (x coordinates)

The change in Northing between a pair of points is referred to as Latitude (not the same as geographic coordinates, but rather – how far North or South between the points.

The change in Easting between a pair of points is referred to as Departure – how far East or West between the points.

# Forward Solution

$$\text{Latitude} = \text{HD} * \text{Cos}(\text{azimuth})$$

$$\text{Departure} = \text{HD} * \text{Sin}(\text{azimuth})$$

Example: If the coordinates of A are 1000.00, 1000.00 (N,E), the Azimuth of line AB is  $83^{\circ}52'30''$  and the horizontal distance AB is 200.615. The coordinates of B will be computed as:

N:	$1000.00 + \{\text{latitude}\}$	$N = 1000.00 + (\text{HD} * \text{Cos}(\text{az})) = 1021.405$
E:	$1000.00 + \{\text{departure}\}$	$E = 1000.00 + (\text{HD} * \text{Sin}(\text{az})) = 1199.470$

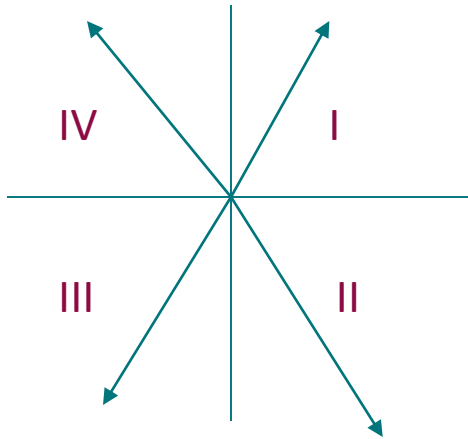
# Inverse Solution

When we have coordinates for a pair of stations, it is often necessary for us to compute the azimuth and horizontal distance between the points – this process is referred to as the Inverse of the coordinates. This is performed in two steps, one for the azimuth and the second for the horizontal distance, as follows:

$$\text{Azimuth AB} = \tan^{-1} \frac{(\text{EastingB} - \text{EastingA})}{(\text{NorthingB} - \text{NorthingA})}$$

However, we must be aware of two important facts: 1) be aware of the units and b) the arctan function will return an angle off of North or South, it is up to you to determine the correct quadrant.

# Inverse Solution



If the Easting of B is greater than A, then the azimuth will be in quadrant I or II, if smaller than A, the azimuth will be in quadrant III or IV

If the Northing of B is greater than A, then the azimuth will be in quadrant I or IV, if smaller than A, the azimuth will be in quadrant II or III.

Rules for correcting angle to Azimuth:

I: Azimuth = angle

II: Azimuth =  $180 - \text{angle}$

III: Azimuth =  $180 + \text{angle}$

IV: Azimuth =  $360 - \text{angle}$

# Inverse Solution

Fortunately, horizontal distance is easier!

$$HD = \sqrt{\text{change}X^2 + \text{change}Y^2}$$

# A look at MS-EXCEL

A number of MS-Excel spreadsheets have been prepared for your use,

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MS-Excel, as a part of the Microsoft Office Suite, is a widely used computational tool that offers precision computations once a few basic skills are achieved. It is available on many platforms: desktop, tablet, android tablets and phones may find free or low cost compatible products.





# Excel examples

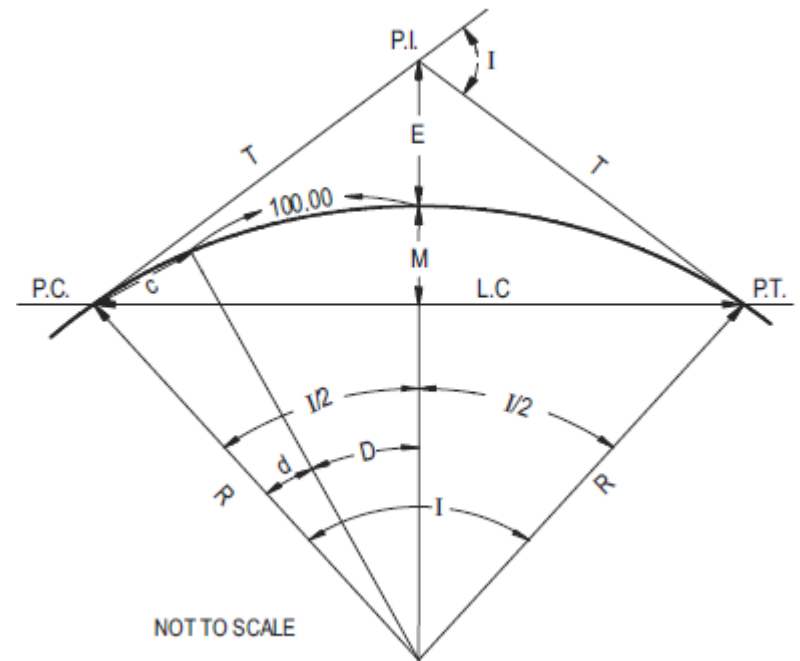
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1. We will begin with the spreadsheet named “AngularUnits”
2. Directions
3. Angles and traverse
4. Level loop

# Horizontal Circular Curves

## HORIZONTAL CIRCULAR CURVES

- D = Degree of curve, arc definition
- L = Length of curve from P.C. to P.T.
- c = Length of sub-chord
- $\ell$  = Length of arc for sub-chord
- d = Central angle for sub-chord



# Circular Curve Formulas

$$D = \frac{5,729.58}{R}$$

$$T = R \tan(I/2)$$

$$L = R I \frac{\pi}{180} = \frac{I}{D}(100)$$

$$LC = 2R \sin(I/2)$$

$$c = 2R \sin(d/2)$$

$$d = \ell D / 100$$

$$M = R [1 - \cos(I/2)]$$

$$E = R \left[ \frac{1}{\cos(I/2)} - 1 \right]$$

$$\text{Area of sector} = \frac{R L}{2} = \frac{\pi R^2 I}{360}$$

$$\text{Area of segment} = \frac{\pi R^2 I}{360} - \frac{R^2 \sin I}{2}$$

These formulas are all derived from two basic geometric identities for circles.

Circumference of the circle =  $2 * \text{Pi} * R$

Area of a circle =  $\text{Pi} * R^2$

# Closing comments?

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Questions?

Door Prizes!



# Acknowledgments

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*Elementary Surveying: An Introduction to Geomatics*, 14<sup>th</sup> ed. by Charles Ghilani  
ISBN: 978-0133758887

NCEES -- Fundamentals of surveying reference book

NSPS Certified Surveying Technician program



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