# Optimization of the LMS Subband, Adaptive Filter System

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**ABSTRACT**

In this paper, we calculate the computational complexity for both fullband and subband adaptive filter systems. This calculation is then used to determine the number of subbands required to minimize computation for given system parameters. Furthermore, certain features of the complexity function are highlighted and a lower bound on the computational reduction realized with subband systems is given.

## 1. INTRODUCTION

The computational requirements for the adaptive adjustment of FIR filters with long impulse responses (on the order of thousands of coefficients) present a formidable computation problem. Even in the case where the adjustment algorithm is simple such as a Least Mean-Square (LMS) algorithm which requires $2N+1$ multiplications and $2N+1$ additions per sample, an adaptive filter with 4000 coefficients operating at a 8kHz sample rate would require over 128 million instruction cycles per second (if one assumes a fused multiply and accumulate (MAC) instruction on the signal processor then the rate can drop to 96 million if implemented properly). Such parameters are typical in the acoustic echo cancellation application illustrated in Figure 1 as an adaptive system modeller.

One technique to overcome the computational problem is to replace the single, fullband adaptive filter, $\hat{\mathbf{h}}$ with multiple, shorter-length subband adaptive filters embedded in a filter bank structure as shown in Figure 5. In the subband adaptive filter system, the desired output signal, $y$, and the input signal, $x$ are split into $M$ subband signals by analysis filters, $\mathbf{f}_0,\cdots,\mathbf{f}_{M-1}$ and downsampled by a factor of $D$. A bank of adaptive filters, $\hat{\mathbf{h}}_0,\cdots,\hat{\mathbf{h}}_{M-1}$ each adjust themselves so as to minimize their expected squared subband error which is taken as the square of the difference between the desired subband signal, $y_m$ and the subband adaptive filter output, $\hat{y}_m$. The subband error signals are used to reconstruct a fullband output [2]. Reconstruction of the fullband error signal, $e$ consists of upsampling the subband error signals, $e_0,\cdots,e_{M-1}$ by a factor of $D$ and filtering with synthesis filters, $\mathbf{g}_0,\cdots,\mathbf{g}_{M-1}$.

The computational benefit of this technique results from the fact that the subband adaptive filters are shorter in length than the fullband adaptive filter (although the total number of FIR coefficients is usually the same) and operate at a downsampled rate. On the downside of this technique is the increased design complexity, end-to-end delay associated with analysis and synthesis filtering, and aliasing associated with downsampling.

## 2. COMPUTATIONAL COMPLEXITY OF THE SUBBAND ADAPTIVE FILTER SYSTEM

The computational complexity of the $M/D$ oversampled, $M$-subband adaptive filter system (Figure 5), will be measured by the number of real multiplications per input sample. In DSP applications such as FIR filtering and FFTs, additions are usually performed in conjunction with multiplications using a MAC instruction. This instruction is vital to many DSP algorithms and counting the number of real multiplications gives a good idea of the number of multiply-accumulate instructions and thus the computational complexity from an implementation perspective. We will assume that $x$ and $y$ are real-valued signals. The computational complexity for the fullband adaptive filter with $N$ coefficients is then

$$C_{\text{fullband}} = 2N + 1. \tag{1}$$

Next the computational complexity for the subband adaptive filter system is derived in two parts: the complexity for analysis and synthesis filtering, $C_{\text{subband},1}$ and the complexity for subband adaptive filtering ($M$ filters), $C_{\text{subband},2}$. For the first part, we assume analysis and synthesis filtering is equivalently and efficiently implemented with the polyphase uniform DFT filter bank (see Figure 6), the prototype lowpass filter is length $L$, and $M/D$ is an integer [3]. Then $C_{\text{subband},1}$ is computed as follows. There are a total of $M$ polyphase filters, each of length $L/D$ operating at a downsampled rate of $1/D$ in the filter bank thus requiring $\frac{LM}{D^2}$ real multiplications per input sample. This operation is performed three times: for the analysis filtering of $x$ and $y$ and for the synthesis filtering of $e_0,\cdots,e_{M-1}$. The $M$-point DFT and IDFT are implemented (assuming $M$ is a power of 2) with a radix-2 FFT which requires $\frac{M}{2}\log_2(M) - M$ complex multiplications. For real data, the $M$-point IDFT can be realized with an $\frac{M}{2}$-point FFT and $M/2$ complex multiplications [4]. This relates to $M\log_2\left(\frac{M}{2}\right)$ real multiplications for the analysis filtering of $x$ and $y$; a similar realization holds for the synthesis filtering of $e_0,\cdots,e_{M-1}$. Thus the total number of real multiplications for subband filtering per input sample is

$$C_{\text{subband},1} = \frac{3LM}{D^2} + 3M\log_2\left(\frac{M}{2}\right). \tag{2}$$

Since the input and desired output signals are real, the DFT is symmetric. Exploiting this symmetry requires processing of $\frac{M}{2}+1$ of the subbands with subbands $\frac{M}{2}+1,\cdots,M-1$, taken as the respective complex conjugates of subbands $\frac{M}{2}-1,\cdots,1$. Furthermore, the uniform DFT bank will yield real signals in subbands 0 and $M/2$ and complex signals in the other subbands. Thus there will be 2 real adaptive filters and $\frac{M}{2}-1$ complex adaptive filters. Assuming the length of each subband adaptive filter is $N$ and operates at the downsampled rate, and the LMS algorithm (either real or complex) is used for the update, the total number of real multiplications for adaptive filtering per input sample is

$$C_{\text{subband},2} = \frac{2(2N+1) + 4\left(\frac{M}{2}-1\right)(2N+1)}{D} = \frac{(2N+1)(2M-2)}{D}\ . \tag{3}$$

The complexity for the subband adaptive filter system is then taken as the sum of (2) and (3)

$$\begin{aligned} C_{\text{subband}} &= C_{\text{subband},1} + C_{\text{subband},2} \\ &= \frac{3LM}{D^2} + \frac{(2N+1)(2M-2)}{D} + 3M\log_2\left(\frac{M}{2}\right). \end{aligned} \tag{4}$$

If the length of the unknown system, **h** to be modeled is $I$, then the normalized ($\frac{C_{\text{subband}}}{C_{\text{fullband}}}$) computational complexity with $C_{\text{fullband}}$ taken from (1) with $N = I$ and $C_{\text{subband}}$ taken from (4) with $N = I/D$ is

$$\begin{aligned} \frac{C_{\text{subband}}}{C_{\text{fullband}}} &= \frac{\frac{3LM}{D^2} + 3M\log_2\left(\frac{M}{2}\right) + \frac{\left(2\frac{I}{D}+1\right)(2M-2)}{D}}{2I+1} \\ &= \frac{\frac{3LM+4I(M-1)}{D^2} + \frac{2(M-1)}{D} + 3M\log_2\left(\frac{M}{2}\right)}{2I+1}\ . \end{aligned} \tag{5}$$

Figure 2 contains a plot of (5) with $I = 4000$ and $L = 64$ versus the number of subbands, $M$ for critically sampled ($D = M$) and $2\times$ oversampled ($D = M/2$) systems. It is clear in this example, that for both critically sampled and $2\times$ oversampled subband systems, there are configurations that require fewer computations than the equivalent fullband system. Since most systems will have subbands sampled between critical and $2\times$ oversampled rates (there do exist efficient polyphase, uniform-DFT filter banks where $M / D$ is not an integer [5]), these curves represent rough lower and upper bounds on normalized computational complexity for the chosen parameters.

## 3. OPTIMAL NUMBER OF SUBBANDS

Given (4), the optimal (in the sense of minimizing the computational complexity) number of subbands, $M$ may be computed. However, since (4) is non-linear, a function of four variables, and $M$ is constrained to be a power of 2 as is the case when using a polyphase uniform DFT filter bank, closed form solutions are difficult to arrive at. Instead, we consider optimization of a system with typical system parameters. The calculation for the optimal number of subbands for other configurations is similar. For the $2\times$ oversampled system with $L = 64$ and $I = 4000$, it can be proven that (4) has one minimum and that the optimal number of subbands (assuming $M$ to be a power of 2) is 64.

The plot in Figure 3 illustrates the optimal number of subbands, $M$ (assuming $M$ is a power of 2) for a $2\times$ oversampled subband adaptive filter system ($L = 64$ and $N = I / D$) versus echo path length. For $184 \le I \le 476$, 16 subbands is optimal, $477 \le I \le 2367$, 32 subbands is optimal, and for $2368 \le I \le 10961$, 64 subbands is optimal. For $I < 184$, the subband system (with above parameters) cannot be designed to be more efficient than the fullband system.

## 4. MONOTONICITY OF THE COMPUTATIONAL COMPLEXITY FUNCTION

From Figure 2 it can be seen that for certain subband adaptive filter system parameters ($M$, $D$, $L$, and $N$), the subband system requires less computation than its fullband equivalent. It will now be shown that as the length of the echo path, $I$ increases, the normalized computational complexity of the subband system monotonically decreases, and thus the curves in Figure 2 will be lower as $I$ increases. Differentiating (5) with respect to $I$ yields

$$\frac{\partial\left(\frac{C_{\text{subband}}}{C_{\text{fullband}}}\right)}{\partial I} = \frac{\frac{4}{D^2}\left[(M-1)(1-D)\right] - \frac{6LM}{D^2} - 6M\log_2\left(\frac{M}{2}\right)}{(2I+1)^2}. \quad (6)$$

For $D \ge 1$, (6) is strictly less than zero thus $\frac{C_{\text{subband}}}{C_{\text{fullband}}}$ monotonically decreases with increasing echo path length. Therefore computational costs associated with implementing a subband adaptive filter system versus the equivalent fullband system decrease with increasing echo path length. The asymptotic value of the normalized computational complexity (5) is computed using L'Hospital's Rule as

$$\lim_{I \to \infty} \frac{C_{\text{subband}}}{C_{\text{fullband}}} = \frac{2}{D^2}(M-1). \quad (7)$$

Figure 4 contains a plot of the normalized computational complexity with $L = 64$ and $M = 64$ versus the length of the echo path, $I$ for critically sampled and $2\times$ oversampled systems. Included in this plot are the asymptotic values for these systems. It can be seen that for a moderately long echo path (~4000 coefficients), a 64-subband, $2\times$ oversampled system provides a 75% computational cost reduction over a similar fullband implementation.

## 5. CONCLUSIONS

In this paper, we have computed the computational complexity (as measured by the number of real multiplies) for a subband adaptive filter system. This calculation demonstrates that there are configurations of the subband system that are more efficient than the equivalent fullband system. For typical system parameters, we have computed the optimal number of subbands to minimize the complexity. We have also presented the optimal number of subbands for a typical system as a function of the echo path length. These optimums are easily computed for other system parameters, Finally, we have shown that as the echo path increases, the subband system's complexity always decreases relative to the fullband system. The asymoptotic value (lower bound) for the normalized computational complexity was given.

## REFERENCES


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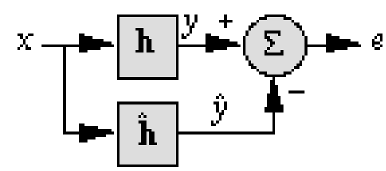


Figure 1: Fullband adaptive system modeller

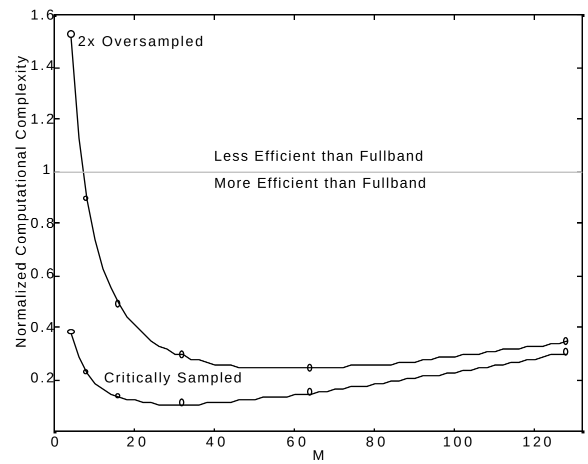


Figure 2: Normalized computational complexity for the subband adaptive filter system versus the number of subbands, *M*.

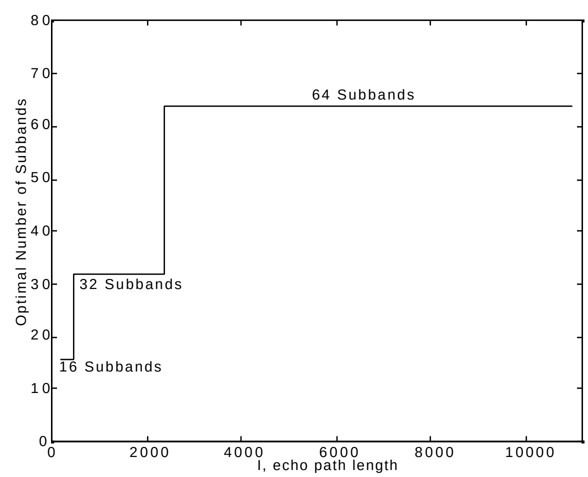


Figure 3: Optimal number of subbands to minimize subband computational complexity as a function of echo path length, *l*.

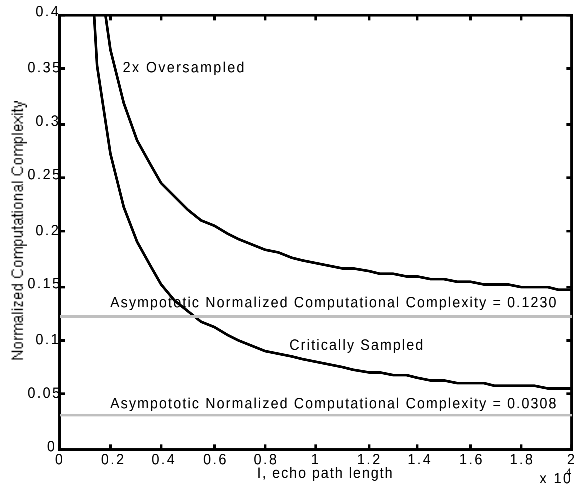


Figure 4: Normalized computational complexity for the 64-subband adaptive filter system as a function of echo path length, $I$.

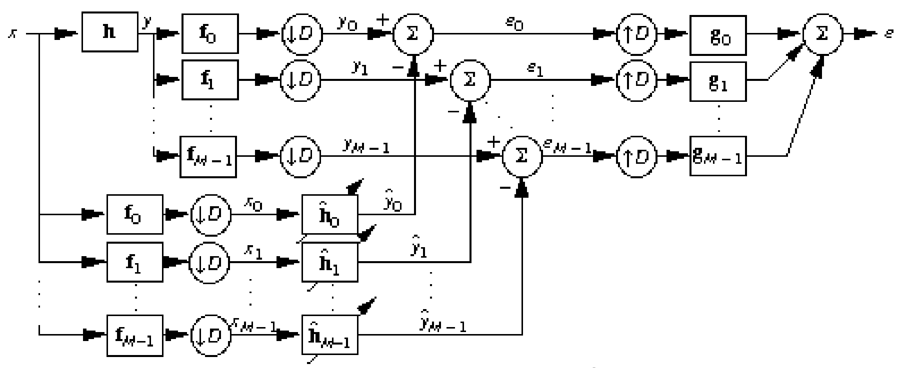

Figure 5: The subband adaptive filter system.

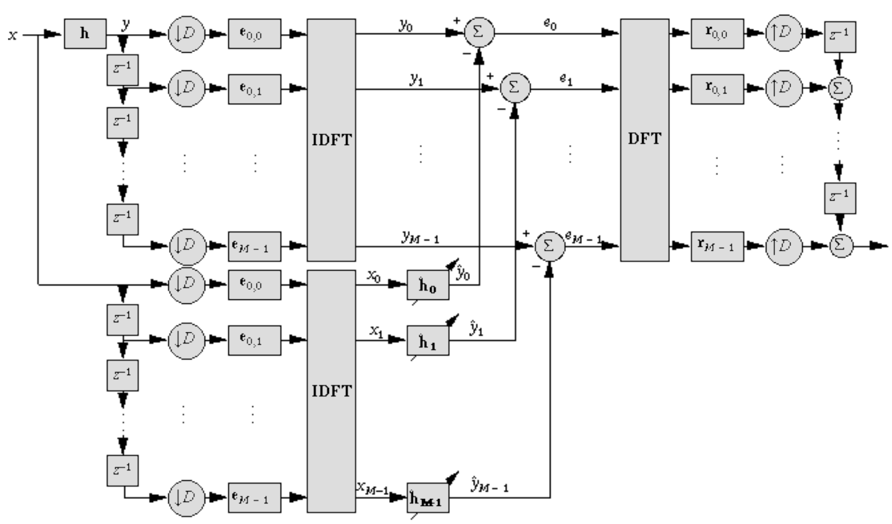


Figure 6: Adaptive filter system with polyphase uniform DFT filter bank