

## ON THE DESIGN OF ANALYSIS/SYNTHESIS FILTERS FOR A 2-CHANNEL, PERFECT RECONSTRUCTION FILTER BANK

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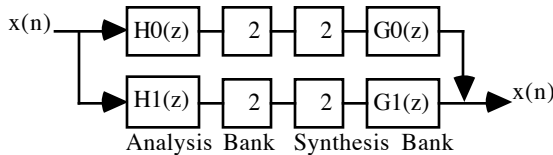
**Abstract--This paper will review a design technique for analysis and synthesis filters used in a 2-channel, perfect reconstruction filter bank. Emphasis will be on actual design of the filters within the filter bank structure using MATLAB.<sup>1</sup>**

### I. INTRODUCTION

A 2-channel filter bank decomposes a signal into two frequency bands enabling the designer to process each signal on a band-by-band basis. This decomposition has proven useful in the past few years in the areas of speech coding and image processing and shows promise in other areas such as adaptive filtering. If the analysis and synthesis filters of the filter bank are not properly designed, the signal can suffer from a variety of ills including aliasing and amplitude and phase distortions. However, these filters may be designed in such a way that the effects of aliasing and the two distortions are eliminated and thus the filter bank has a perfect reconstruction property (i.e. the output is a scaled and delayed version of the input). In this paper we offer a short review on the theory of perfect reconstruction filter banks for the 2-channel case and present our results for actual design and implementation of these filter banks using MATLAB.

### II. DESIGNING ANALYSIS AND SYNTHESIS FILTERS FOR A 2-CHANNEL PERFECT RECONSTRUCTION FILTER BANK

The typical 2-channel filter bank, often referred to as a quadrature mirror filter bank (QMF), is illustrated in Fig. 1:



**Figure 1:** 2-Channel Filter Bank

The decomposition of  $x(n)$  into two frequency bands is performed in the analysis bank by  $H_0(e^{j\omega})$  (designed to be an order  $N-1$ , low-pass filter) and  $H_1(e^{j\omega})$  (designed to be an

overlapping, complimentary high-pass filter). The respective output signals are then critically decimated. In the synthesis bank, the signals are interpolated, filtered by the synthesis filters, and recombined to yield a reconstructed signal. The overlapping feature of the analysis filters allows the designer to relax the sharp cutoff requirements at the expense of introducing aliasing into the system. As Smith and Barnwell [1] have shown, this aliasing can be canceled by proper design of the synthesis filters.

A frequency-domain analysis of the reconstructed signal yields the following result:

$$X(e^{j\omega}) = \frac{1}{2} \begin{bmatrix} G_0(e^{j\omega}) & G_1(e^{j\omega}) \end{bmatrix} \begin{bmatrix} H_0(e^{j\omega}) & H_0(-e^{j\omega}) \\ H_1(e^{j\omega}) & H_1(-e^{j\omega}) \end{bmatrix} \begin{bmatrix} X(e^{j\omega}) \\ X(-e^{j\omega}) \end{bmatrix} \quad (1)$$

The term corresponding to  $X(-e^{j\omega})$  represents the effects of aliasing and imaging due to the decimators and interpolators. The aliasing term may be canceled by designing the synthesis filters under the constraint that:

$$G_0(e^{j\omega}) = H_1(-e^{j\omega}) \quad (2a)$$

$$G_1(e^{j\omega}) = -H_0(-e^{j\omega}) \quad (2b)$$

We call (2) the alias cancellation condition. This choice forces the aliasing term to equal 0, resulting in a system transfer function,

$$C(e^{j\omega}) = \frac{X(e^{j\omega})}{X(-e^{j\omega})} = \frac{1}{2} [H_0(e^{j\omega})H_1(-e^{j\omega}) - H_1(e^{j\omega})H_0(-e^{j\omega})] \quad (3)$$

In order that the 2-channel filter bank have the perfect reconstruction property, (2) must hold and  $C(e^{j\omega}) = ce^{-j\omega(N-1)}$ . This latter condition implies that:

$$H_0(e^{j\omega}) H_1(-e^{j\omega}) - H_1(e^{j\omega}) H_0(-e^{j\omega}) = de^{-j\omega(N-1)} \quad (4)$$

where  $d$  is a constant. In order to achieve (4), Smith and Barnwell [1] define the channel 1 analysis filter to be:

$$H_1(e^{j\omega}) = -e^{-j\omega(N-1)}H_0(-e^{-j\omega}) \quad (5)$$

This definition implies that:

$$H_0(e^{j\omega}) H_0(e^{-j\omega}) + H_0(-e^{j\omega}) H_0(-e^{-j\omega}) = d \quad (6)$$

<sup>1</sup>This work was funded in part by AT&T Bell Laboratories and the Air Force Office of Scientific Research.

Therefore, we design an FIR linear phase, half-band filter  $F(e^{j\omega})$  and perform a spectral factorization so that:

$$F(e^{j\omega}) = e^{-j\omega(N-1)}H_0(e^{j\omega})H_0(e^{-j\omega}) \quad (7)$$

Note that  $N-1$  must be odd or severe amplitude distortion will result, see [2]. We then apply (2) and (6) or the equivalent time-domain expressions:

$$h_1(n) = (-1)^n h_0(N-n) \quad (8a)$$

$$g_0(n) = h_0(N-n) \quad (8b)$$

$$g_1(n) = h_1(N-n) \quad (8c)$$

to determine the remaining filters in the filter bank. With these filters, the output of the 2-channel filter bank is simply a delayed and scaled version of the input. There is some flexibility as to which zeros to assign to  $H_0(e^{j\omega})$  and

$H_0(e^{-j\omega})$ . For simplicity, we take  $H_0(e^{j\omega})$  to be minimum phase i.e. all zeros are inside or on the unit circle. For alternate assignments see [1].

### III. DESIGN ALGORITHM FOR A 2-CHANNEL PERFECT RECONSTRUCTION FILTER BANK

The following algorithm is outlined in [2]:

1. Design an order  $2(N-1)$  ( $N-1$  odd), FIR half-band filter  $F(e^{j\omega})$  using any of the common design algorithms such as Parks-McClellan. The half-band property can be achieved by constraining the passband and stopband edges to be such that  $\omega_p + \omega_s = \pi$  and the peak ripple in the passband and stopband are equal.
2. Define  $F_+(e^{j\omega}) = F(e^{j\omega}) + \delta$ , where  $\delta$  is the ripple in  $F(e^{j\omega})$ .
3. Spectrally factor  $F_+(e^{j\omega}) = e^{-j\omega(N-1)}H_0(e^{j\omega})H_0(e^{-j\omega})$ .
4. Compute  $H_1(e^{j\omega})$ ,  $G_0(e^{j\omega})$ , and  $G_1(e^{j\omega})$  via (2) and (6) or (8).

The following example will make the algorithm more clear and illuminate some design issues that occur:

This example calls for  $H_0(e^{j\omega})$  to be a low pass FIR of order 15 and transition width to be  $0.12\pi$ . We begin by designing an order 30 half-band filter  $F(e^{j\omega})$  with  $\omega_p = 0.44\pi$ ,  $\omega_s = 0.56\pi$  using MATLAB's implementation of the Parks-McClellan algorithm (see Fig. 2):

```
N = 16;
wp = 0.44;
ws = 0.56;
f_frequency = [0.0 wp ws 1.0];
f_magnitude = [1.0 1.0 0.0 0.0];
f = remez((2*(N-1)), f_frequency, f_magnitude);
```

Next, we must make the amplitude response of  $F(e^{j\omega})$  non-negative (see Fig. 3):

```
fplus = f;
ripple = max(abs(f_frequency_value)) - 1;
fplus(N) = fplus(N) + ripple;
```

Next, we factor  $F_+(e^{j\omega})$  and sort the roots in ascending order according to magnitude (for min phase assignment). In this example, MATLAB places (for each double zero) one zero in the top half of the vector `fplus_roots` and the other in the bottom half of the vector so that we can assign the top half of the vector (which contains all zeros inside the unit circle and one of each double zero) to  $H_0(e^{j\omega})$ . It is a good idea to make sure that  $H_0(e^{j\omega})$  contains the proper zeros (see Figs. 4(a) and 4(b)).

```
fplus_roots = sort(fplus_roots);
h0 = poly(fplus_roots(1:(N-1)));
h0 = real(h0);
```

We next scale  $H_0(e^{j\omega})$  for unit magnitude in passband (see Fig. 5):

```
[h0_frequency_value, frequency] =
freqz(h0, 1, 512);
range = floor((512/2) - ((0.5 - wp)*512));
scale_factor =
mean(abs(h0_frequency_value(1:range)));
h0 = h0 / scale_factor;
```

Finally, the remaining filters are assigned (see Fig. 6):

```
h1 = h0(N:-1:1);
h1(2:2:N) = (-1) * h1(2:2:N);
g0 = h0(N:-1:1);
g1 = h0;
g1(1:2:N) = (-1) * g1(1:2:N);
```

Verification that these filters induce the perfect reconstruction property (i.e. the system impulse response =  $\delta(k-N+1)$ ) can be computed with the following (see Fig.7):

```
h0(2:2:16) = zeros(1,8);
h1(2:2:16) = zeros(1,8);
y0 = conv(h0, g0);
y1 = conv(h1, g1);
output = y0 + y1;
plot(output)
```

### IV. CONCLUSION

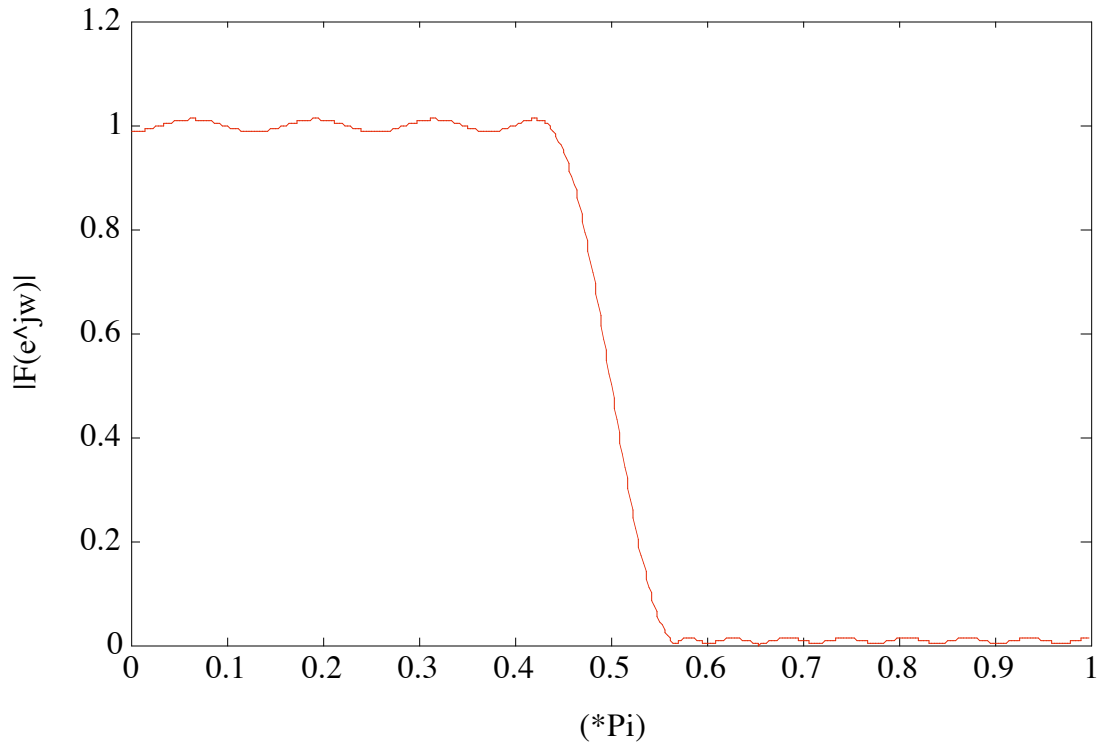
In this paper, we have reviewed a technique for the design of the analysis and synthesis filters used in a 2-channel, perfect reconstruction filter bank. We presented a very simple and efficient method to design these filters using MATLAB.

### REFERENCES

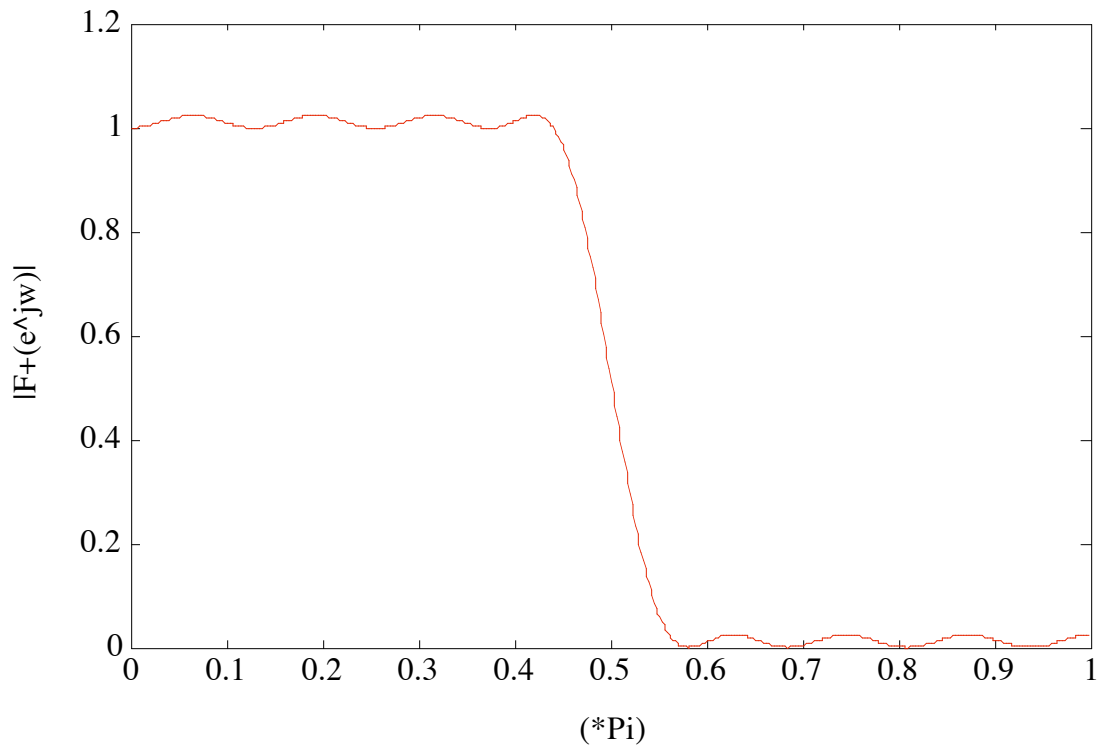
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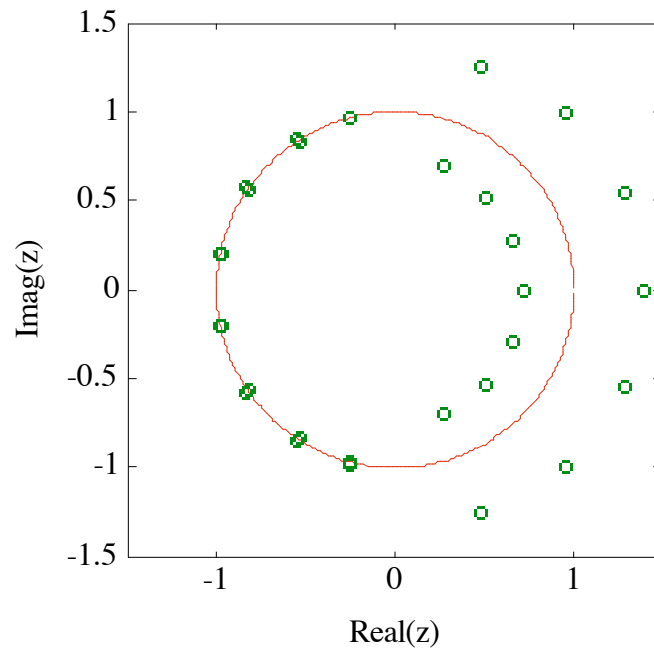
- [2] P.P. Vaidyanathan, "Quadrature Mirror Filter Banks, M-Band Extensions and Perfect-Reconstruction Techniques," IEEE ASSP Magazine, pp.4-20, July 1987.



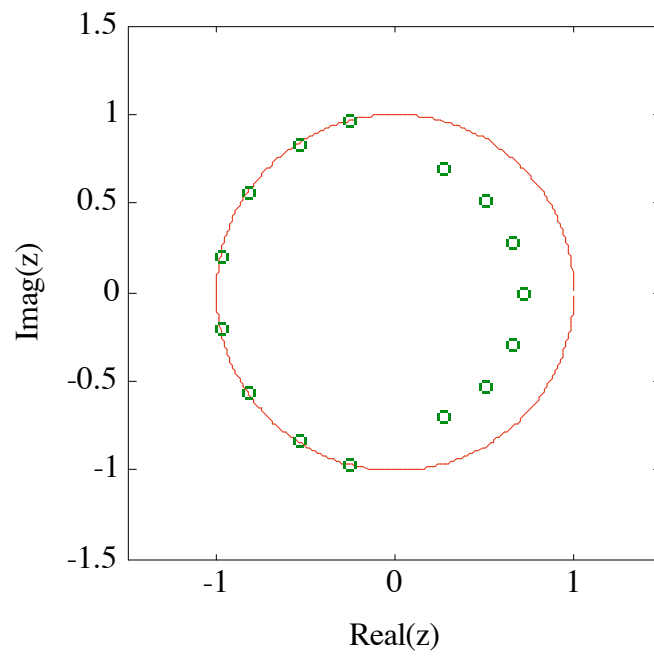
**Figure 2:** Magnitude Response of  $F(e^{j\omega})$



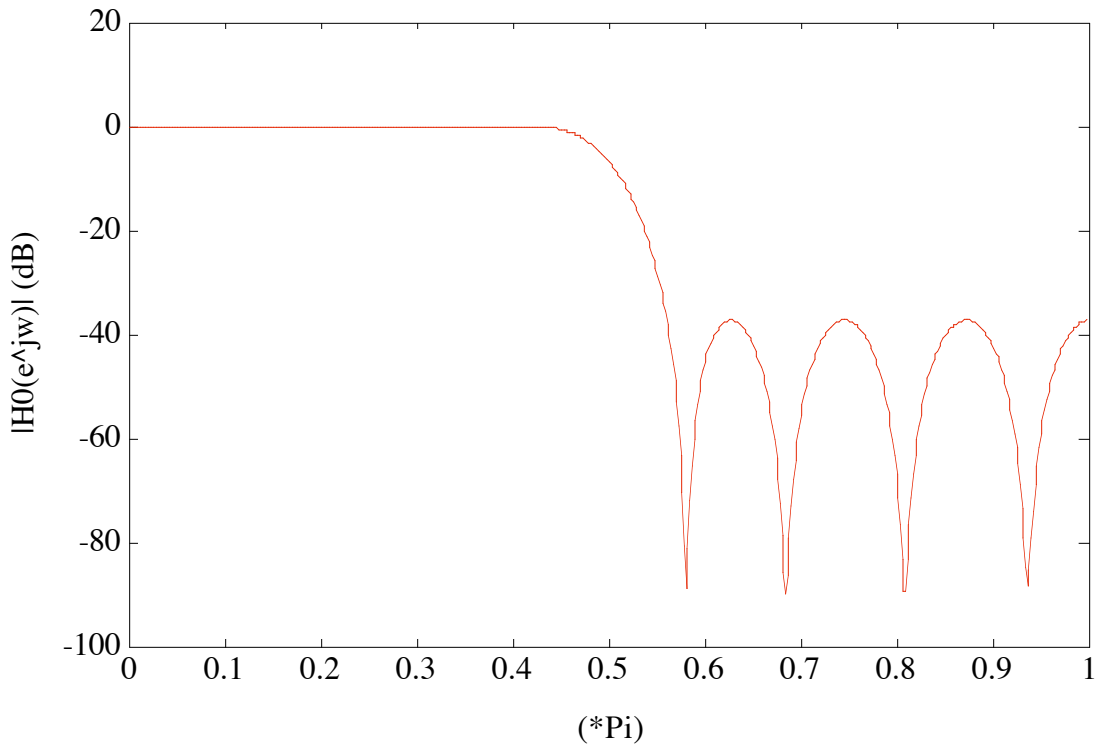
**Figure 3:** Magnitude Response of  $F(e^{j\omega}) + \delta$



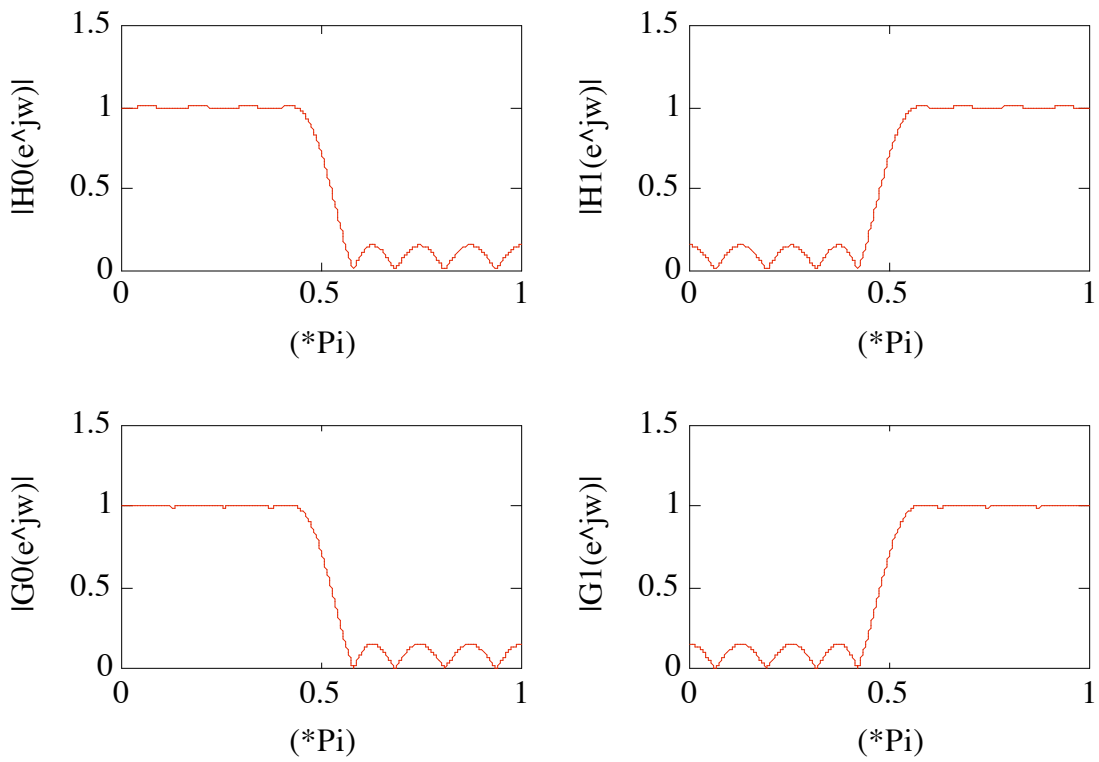
**Figure 4(a):** Roots of  $F(z)$



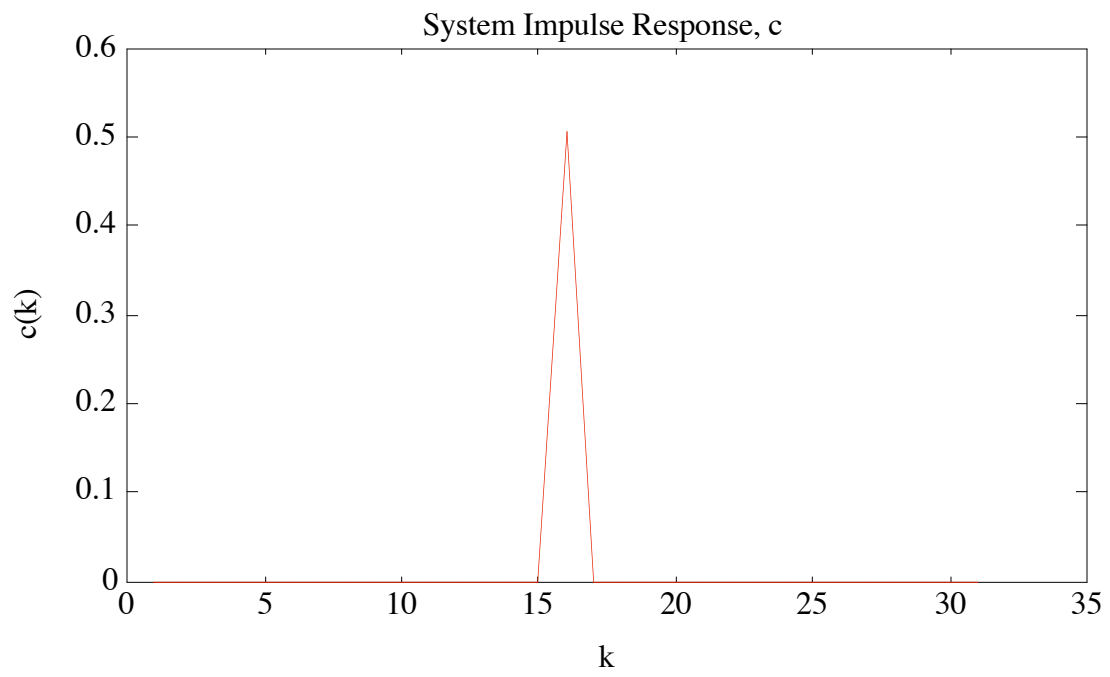
**Figure 4(b):** Roots of min-phase  $H_0(z)$  Filter



**Figure 5:** Channel 0 Analysis Filter



**Figure 6** (left to right, top to bottom): Channel 0 Analysis Filter, Channel 1 Analysis Filter, Channel 0 Synthesis Filter, Channel 1 Synthesis Filter



**Figure 7:** System Impulse Response for Design Example